

Panel 1

# ES204 Mechanical Systems

## Relative Motion Lecture 02

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Dr. Fisher

Panel 2

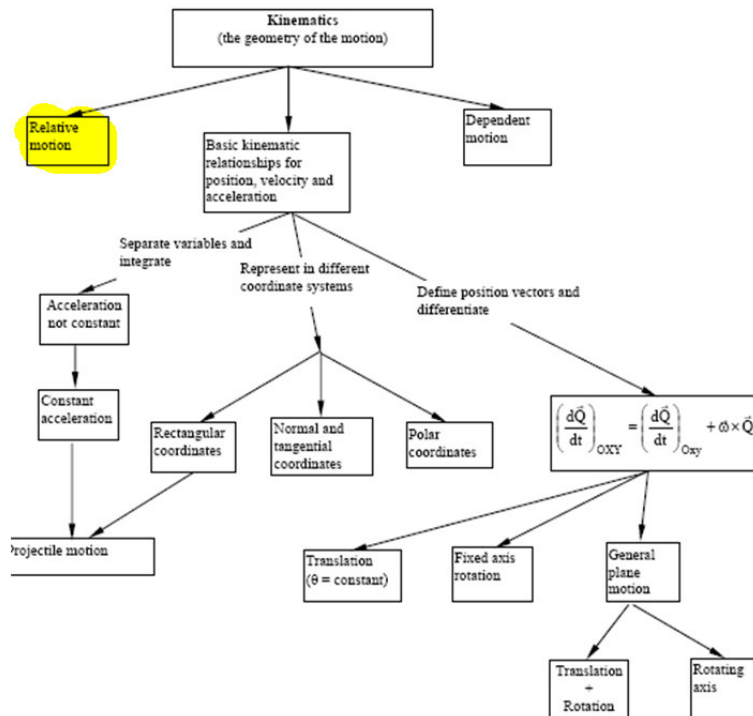
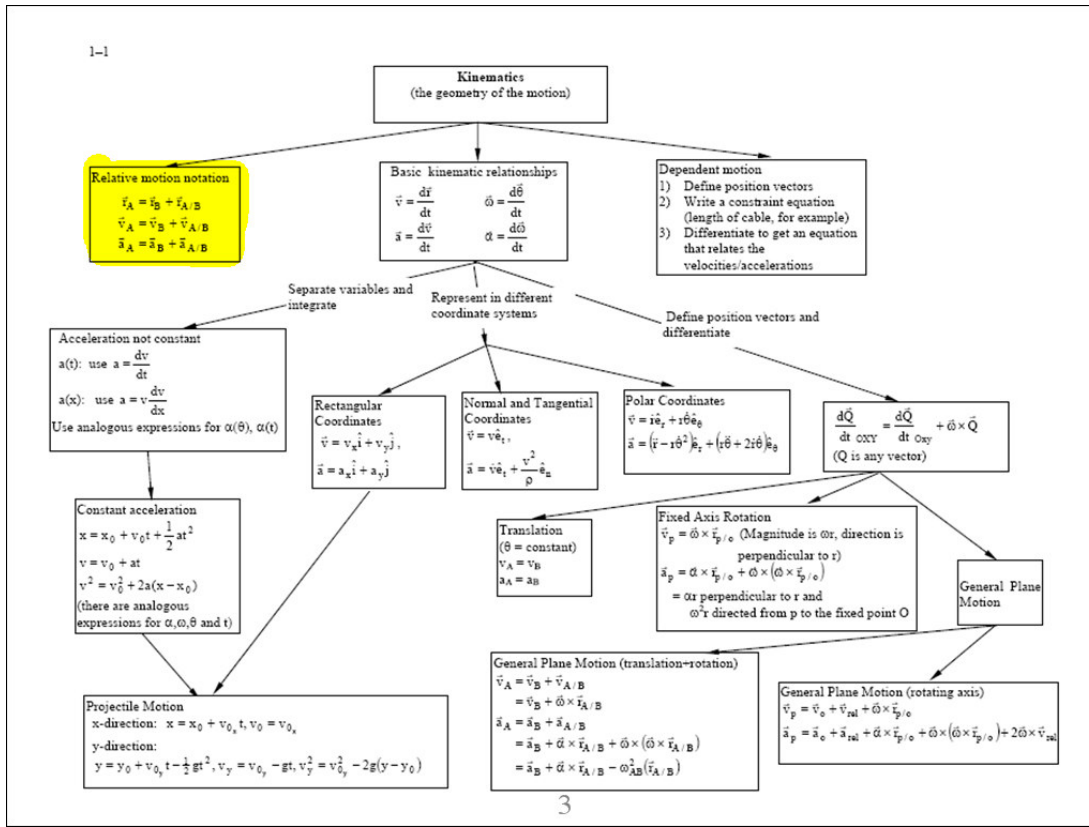


Figure 1.5 Concept Map for Kinematics

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Panel 3



Panel 4

## Relative Motion

**All motion is relative. We have to have some frame of reference (coordinate system) in order to measure position, velocity and acceleration.**

Two groups:

- 1) Fixed or Inertial
  - a. Don't actually exist
  - b. Acceleration is small enough we can consider it to be fixed.
- 2) Moving reference systems

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Relative motion

$O = \text{origin of fixed coordinate system}$   
 $A = \text{origin of moving coordinate system}$   
 $B = \text{another moving point}$

(1)

Note: This is a vector equation. We can

1. Draw vector triangles
2. Write as components and equate components.

Differentiate Eq. 1

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \quad (2)$$

Differentiate Eq. 2.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \quad (3)$$

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**Example:** What is the direction of the velocity of car B with respect to car A?

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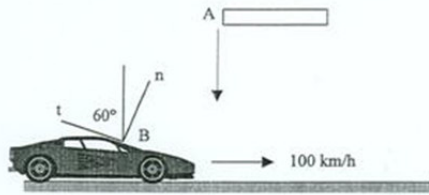
**Example 2**

A drop of water falls with no initial speed from point A of a highway overpass. After dropping 6 m, it strikes the windshield of a passing car that is traveling at a speed of 100 km/h on a horizontal road. If the windshield is inclined  $60^\circ$  from the vertical as shown, determine the angle  $\theta$  relative to the normal  $n$  to the windshield at which the water drop strikes.

Find  $\vec{V}_{\text{Drop/Car}}$

Start by finding

$\vec{V}_{\text{car}}$  &  $\vec{V}_{\text{Drop}}$

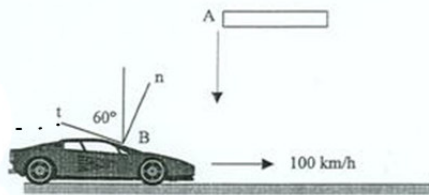


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**Example 2**

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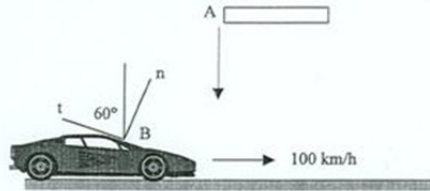
**Solution Sheet**



Panel 9

**Example 2**

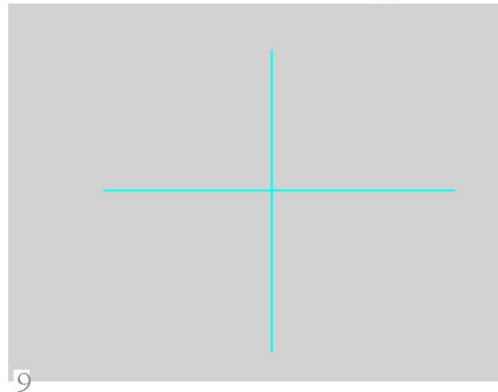
A drop of water falls with no initial speed from point A of a highway overpass. After dropping 6 m, it strikes the windshield of a passing car that is traveling at a speed of 100 km/h on a horizontal road. If the windshield is inclined 60° from the vertical as shown, determine the angle  $\theta$  relative to the normal  $n$  to the windshield at which the water drop strikes.



$$\vec{V}_{car} = 27.8 \frac{m}{s} \hat{i}$$

$$\vec{V}_{drop} = -10.85 \frac{m}{s} \hat{j}$$

Draw relative motion diagram



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Find  $\vec{V}_{Drop/Car}$

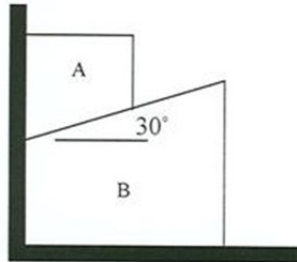
- ☾
- ☾
- ☾
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The raindrop strikes the windshield at an angle of 38.7 degrees compared to the normal direction of the windshield

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**Known :** The two blocks shown have very smooth surfaces. They are held in the configuration shown and at time zero are suddenly released.

**Find :** What is the initial acceleration of each block?



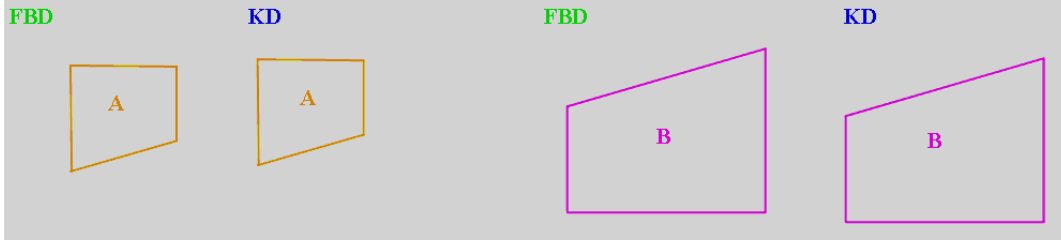
**Given :** The mass of block A is 20 kg and the mass of block B is 50 kg. Friction can be neglected.

**Strategy :** We want to use the rate form of the *conservation of linear momentum principle*. This form will have the desired acceleration terms and the applied forces of weight and the wall/floor reactions. The big question is what to choose as the system: both blocks together or each block separately? Unfortunately, the answer is not readily obvious. If we consider the two blocks together, there is no easy way to inform our model that there is no friction between them. We therefore need to examine both blocks separately, being sure to remember that the contact force between the blocks is perpendicular to the contact surface *and* equal and opposite. Having applied the kinetic equations, we may need to use the kinematic relationship of relative acceleration to get enough equations to solve for our unknowns.

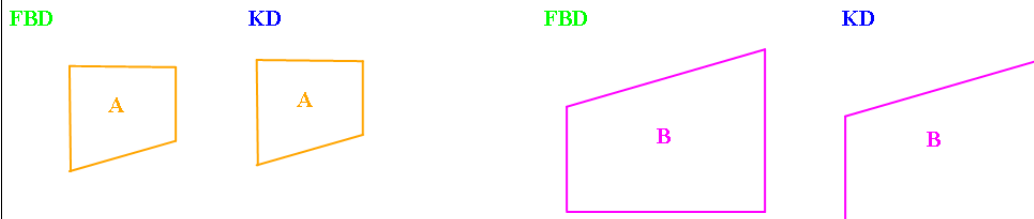
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Your efforts prior to seeing solution



Corrected Solution



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Panel 13

$\sum \frac{d\vec{p}}{dt} = \sum \vec{F}$

X-dir rate equation (Block A)

X-dir rate equation (block B)

Y-dir rate equation (Block A)

Y-dir rate equation (block B)

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Panel 14

What is the direction of  $\vec{a}_{b/a}$ ?

Eq	Unknowns
1	
2	
3	
4	

Kinematic equation.

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Panel 15

Free-body diagrams for blocks A and B. Block A is on an inclined plane with forces  $N_A$ ,  $mg$ , and  $N_B$ . Block B is on a horizontal surface with forces  $N_B$  and  $m_b a$ .

Equations of motion:

$$\sum \vec{F}_{sys} = \Sigma F$$

$$(x) -N_{AB} \cos 60^\circ + N_A = 0$$

$$(y) N_{AB} \sin 60^\circ - m_B g = -m_A a_A$$

$$(y) -N_{AB} \cos 30^\circ - m_B g + N_B = 0$$

$$(x) N_{AB} \sin 30^\circ = m_b a_b$$

Relationship between accelerations:

$$\tan \theta = \frac{a_A}{a_B}$$

Eq	Unknowns
1	$N_A$
2	$N_B$
3	$N_{AB}$
4	$a_A$
5	$a_B$

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**Example problem**

- >  $m_A := 20$ ;
- >  $m_B := 50$ ;
- >  $\theta := 30^\circ \text{Pi} / 180$ ;
- >  $g := 9.81$ ;

**equations**

- >  $eq1 := N_A - N_B \sin(\theta) = 0$ ;
- >  $eq2 := m_A g - N_B \cos(\theta) = m_A a_A$ ;
- >  $eq3 := N_B \sin(\theta) = m_B a_B$ ;
- >  $eq4 := N_B - m_B g - N_B \cos(\theta) = 0$ ;
- >  $eq5 := a_A = a_B \tan(\theta)$ ;

$$eq1 := N_A - \frac{1}{2} N_B = 0$$

$$eq2 := 196.20 - \frac{1}{2} N_B \sqrt{3} = 20 a_A$$

$$eq3 := \frac{1}{2} N_B = 50 a_B$$

$$eq4 := N_B - 490.50 - \frac{1}{2} N_B \sqrt{3} = 0$$

$$eq5 := a_A = \frac{1}{3} a_B \sqrt{3}$$

> **solve**({eq1,eq2,eq3,eq4,eq5});

{  $a_A = 1.154117647$ ,  $N_B = 663.6176471$ ,  $N_{AB} = 199.8990402$ ,  $a_B = 1.998990402$ ,  $N_A = 99.94952011$  }

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