Problem 4.48 **Crane Problem – Analytical Solution**

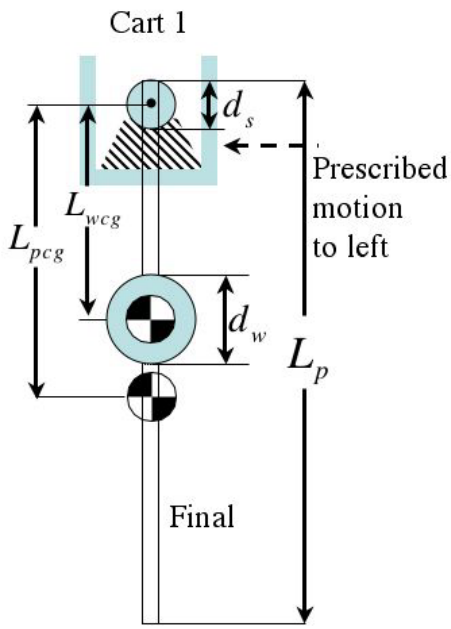

Figure 1: System Schematic

The pendulum / cart system is used to model the motion of an overhead crane. If the cart is given a specified acceleration profile of

$$b(t)=\frac{3597e^{x}\left(1-e^{x}\right)}{\left(1+e^{x}\right)^{3}}+\frac{3597e^{y}\left(1-e^{y}\right)}{\left(1+e^{y}\right)^{3}}\ \text{cm/s}^2$$

Where

$$x=-43.5668t+9.38$$

$$y = -43.5668t + 31.1633$$

and $0 < t < 0.9$,

Determine the best location of the moveable mass L_{wcg} such that the pendulum angular velocity is zero at $t = 0.9$ seconds. Note that there will be an unknown force acting in the direction of the cart motion so that the cart experiences the specified acceleration.

pendulum mass	m_p	= 68.5 g
moveable mass	m_{add}	= 88.0 g
Pendulum length	L_p	= 43.2 cm
sensor diameter	d_s	= 2.5 cm
moveable weight diameter	d_w	= 5.0 cm
pivot to moveable weight cg	L_{wcg}	
pivot to pendulum cg	L_{pcg}	

Notes:

- Draw the system in a displaced orientation to obtain the governing differential equation. (This is the most important part of the analysis part of this lab).
- Write your resulting non-linear differential equation as:

$$\left[something\right]\ddot{\theta}+\left[something\ else\right]sin(\theta)=\left[something\ different\right]b(t)cos(\theta)$$

When implementing this equation in Maple it should look something like this:

diff_eq:=(IGp+IGw+mp*LGp^2+mw*Lw^2)*diff(theta(t),t$2)+(mp*LGp+mw*Lw)*g*sin(theta(t))= … (right hand side missing on purpose – we don't want to give you the whole answer!)

- Maple can numerically solve the non-linear differential equation of motion using:
  >soln := dsolve({diff_eq, theta(0)=0, D(theta) (0)=0}, theta(t),numeric);

  To plot your solution use
  >odeplot(soln, [t,D(theta)(t)], 0..2, numpoints=300);

  Be sure to include >with(plots): at the beginning of your Maple Worksheet. Once your Maple worksheet is working correctly you can vary Lw to find the location where the angular velocity is zero at the end of the acceleration.