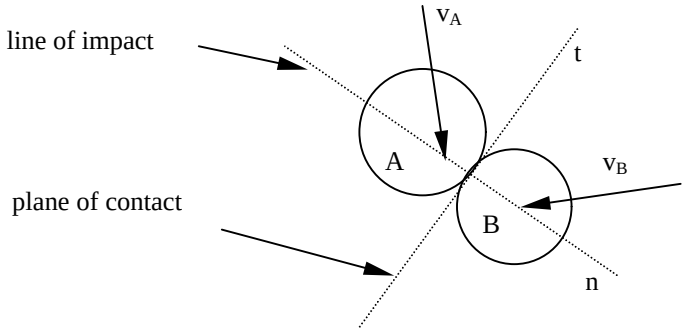
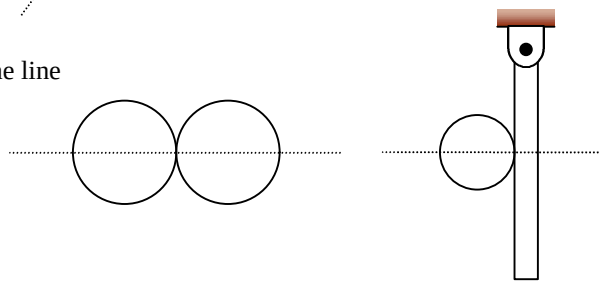
Impact / Coefficient of Restitution

Sketch of impact

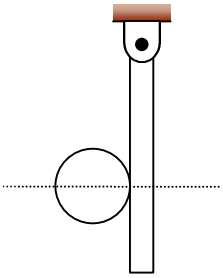


Definitions

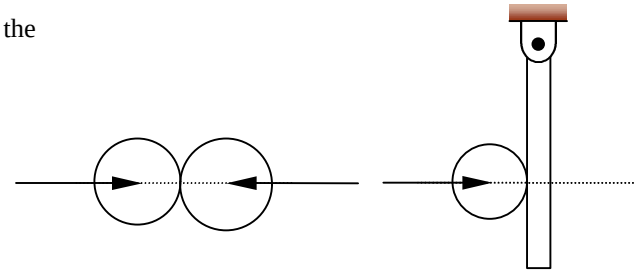
Central Impact: Both mass centers are located on the line of impact (always true for particles).



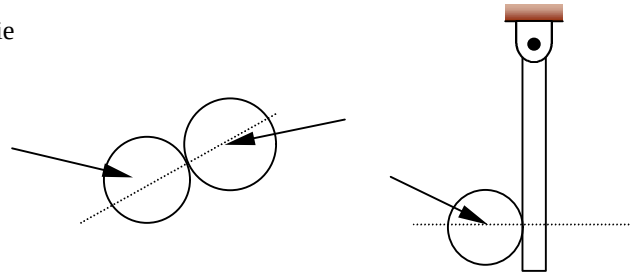
Eccentric Impact: Both mass centers are not located on the line of impact.



Direct Impact: Both velocities, v_A and v_B, lie along the line of impact.



Oblique Impact: Both velocities, v_A and v_B, do not lie along the impact.



Procedure:

1. Draw the impact (impulse-momentum diagrams are especially helpful for rigid body impact).
2. Look at each object by itself, that is, define your system to be each object individually first.
   - Is there any direction with no impulsive forces acting? If so, apply LM in that direction. Note: A direction that frequently does not have any impulses acting is along the plane of contact (tangential direction).
   - Is there any point that you could take angular momentum about that has no impulsive moments? If so, apply AM about that point.
3. Look at both objects together, that is, define your system to be both objects.
   - Is there any new direction with no impulsive forces acting? If so, apply LM in that direction.
   - Is there any point that you could take angular momentum about that has no impulsive moments? If so, apply AM about that point.
4. **Coefficient of restitution** (experimentally determined *constitutive model* for a given combination of materials, assumed to be constant)

$$e = -\left(\frac{v'_{PB_n} - v'_{PA_n}}{v_{PB_n} - v_{PA_n}}\right) \quad \text{or} \quad v'_{PB_n} - v'_{PA_n} = -e\left(v_{PB_n} - v_{PA_n}\right) = e\left(v_{PA_n} - v_{PB_n}\right)$$

where

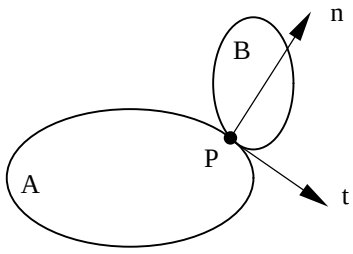
e = the coefficient of restitution $(0 \le e \le 1)$

v'_{PA_n} = the velocity of the point of contact on object A in the normal direction after the impact

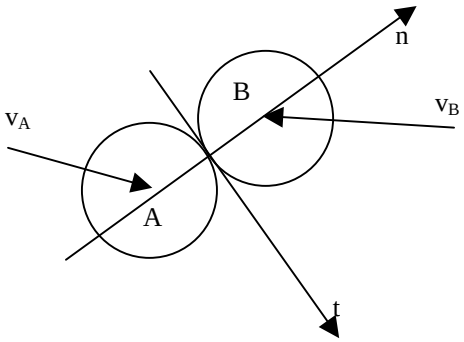
v'_{PB_n} = the velocity of the point of contact on object B in the normal direction after the impact

v_{PA_n} = the velocity of the point of contact on object A in the normal direction before the impact

v_{PB_n} = the velocity of the point of contact on object B in the normal direction before the impact



Special Case: Oblique Central Impact



A in t-direction: $v'_{At} = v_{At}$

B in t-direction: $v'_{Bt} = v_{Bt}$

A and B together in n-direction: $m_A v_{An} + m_B v_{Bn} = m_A v'_{An} + m_B v'_{Bn}$

Coefficient of Restitution: $v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$