ROSE-HULMAN INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

Example Problem - Le28

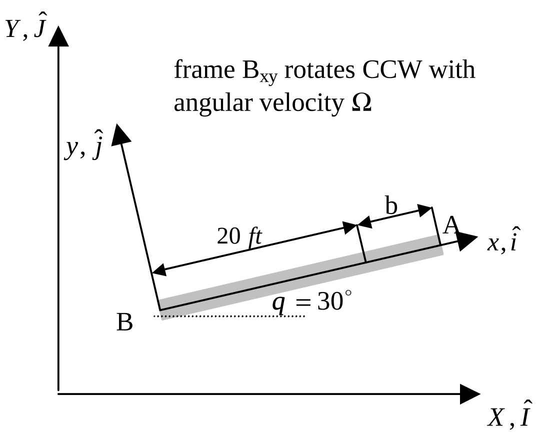
2. The fire truck is moving forward at a speed of 35 mi/hr (51.33 ft/sec) and is decelerating at the rate of 10 ft/sec^2. Simultaneously, the ladder is being raised and extended. At the instant considered, the angle is 30° and is increasing at the constant rate of 10 deg/sec. Also, at this instant the extension *b* of the ladder is 5 ft, with an extension rate of 2 ft/sec and an extension acceleration of -1 ft/sec^2.

Determine:

1. The global velocity and acceleration of point A
2. The reaction at B if the weight of the ladder is 100 pounds and it is modeled as a homogenous slender rod. Replace the piston with an applied moment at B.

Part 1:Velocity and Acceleration of A

Step 1: Identify System - *the ladder*
Step 2: Identify Form of Equations Required - the problem is *pure kinematics* and will require *rotating frames*. Point B is a great choice for the origin of the local reference frame.
Step 3: Draw system diagrams and **identify unknowns**.



At the instant shown :

$$b = 5\,ft$$
$$\dot{b} = 2\ ft/s$$
$$\ddot{b} = -1\ ft/s^2$$
$$\Omega = 10^\circ/s = \frac{p}{18}\,rad/s$$
$$\dot{\Omega} = 0$$
$$\bar{v}_B = 51.33\ ft/s\ \hat{I}$$
$$\bar{a}_B = -10\ ft/s^2\ \hat{I}$$

Step 4: Kinetics
Part 1 is pure kinematics so kinetics are not required

Step 5: Kinematics

Frame Coordinate Transformation Matrix

$$\begin{bmatrix}\hat{i}\\ \hat{j}\end{bmatrix} = \begin{bmatrix}\cos q & \sin q\\ -\sin q & \cos q\end{bmatrix}\begin{bmatrix}\hat{I}\\ \hat{J}\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}\hat{I}\\ \hat{J}\end{bmatrix}$$

Rotating Frame relative velocity of A wrt B

$$\bar{v}_A = \bar{v}_B + \bar{\Omega} \times \bar{r}_{A/B} + \left(\bar{v}_{A/B}\right)_{xy}$$

Expanding and collecting I,i and J,j components

I, i : $$v_{A_X}\hat{I} = v_{B_X}\hat{I} - \Omega r_{A/B_y}\hat{i} + \left(v_{A/B}\right)_{xy_x}\hat{i} \quad (1)$$

J, j : $$v_{A_Y}\hat{J} = v_{B_Y}\hat{J} + \Omega r_{A/B_x}\hat{j} + \left(v_{A/B}\right)_{xy_y}\hat{j} \quad (2)$$

Substituting our knowns

I, i : $$v_{A_X}\hat{I} = 51.33\hat{I} - \frac{p}{18}0\hat{i} + 2\hat{i} = 51.33\hat{I} + 2\hat{i} \quad (3)$$

J, j : $$V_{A_Y}\hat{J} = 0\hat{I} + \frac{p}{18}25\hat{j} + 0\hat{j} = \frac{25p}{18}\hat{j} \quad (4)$$

Combining (3) and (4)

$$\bar{v}_A = 51.33\hat{I} + \left(2\hat{i} + \frac{25p}{18}\hat{j}\right) \quad (5)$$

Note that we have *mixed frames* - applying the coordinate transformation matrix to convert the local velocities to global velocities.

$$2\hat{i} + \frac{25p}{18}\hat{j} \Rightarrow \begin{bmatrix} 2 & 4.36 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \hat{I} \\ \hat{J} \end{bmatrix} = -1.61\hat{I} - 0.734\hat{J}$$

Substituting into (5)

$$\boxed{\bar{v}_A = 50.9\hat{I} + 4.78\hat{J}}$$

Rotating Frame relative acceleration of A wrt B

$$\bar{a}_A = \bar{a}_B + \dot{\bar{\Omega}} \times \bar{r}_{A/B} + \bar{\Omega} \times \left(\bar{\Omega} \times \bar{r}_{A/B} \right) + 2\bar{\Omega} \times \left(\bar{v}_{A/B} \right)_{xy} + \left(\bar{a}_{A/B} \right)_{xy}$$

Expanding and collecting I,i and J,j components

I, i :

J, j :

Substituting our knowns

I, i :

J, j :

Combining

Resolving the mixed frames

Thus the acceleration is