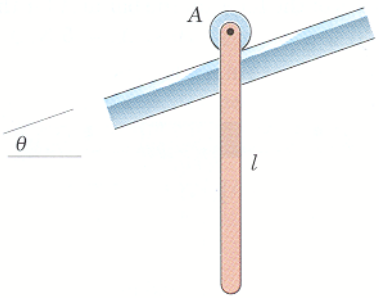
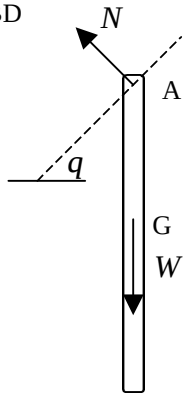
Example Problem - Le 24

6/97 The slender rod of mass m and length l is released from rest in the vertical position with the small, i.e. negligible, roller at end A resting on the incline. Determine the initial acceleration of A. (*taken from Dynamics, 4th Edition by Meriam & Kraige*)

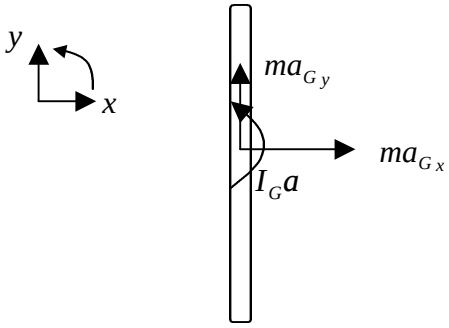


Strategy: Isolate system (rod and roller)
Kinetics (COLM, COAM RF)
Kinematics (relative acceleration)

FBD



KD



unk	eqs
a_A	1
N	2
W	3
a_{Gx}	4
a_{Gy}	5
I_G	6
α	7
a_{Ax}	8
a_{Ay}	9
ω	10

Kinetics:

COLM (RF)

x-dir: $$-N\sin q = ma_{G_x} \quad (1)$$

y-dir: $$N\cos q - W = ma_{G_y} \quad (2)$$

COAM (RF) about A

$$0 = I_G a + ma_{G_x}\left(\frac{l}{2}\right) \quad (3)$$

Kinematics:

Relate the acceleration at A to the acceleration at G

$$\bar{a}_A = \bar{a}_G + \bar{a}_{A/G} = \bar{a}_G + \bar{\alpha}\times\bar{r}_{A/G} - w^2\bar{r}_{A/G}$$

$$a_{A_x}\hat{i} + a_{A_y}\hat{j} = a_{G_x}\hat{i} + a_{G_y}\hat{j} + \left(a\hat{k}\right)\times\left(r_{A/G_x}\hat{i} + r_{A/G_y}\hat{j}\right) - w^2\left(r_{A/G_x}\hat{i} + r_{A/G_y}\hat{j}\right)$$

$$= a_{G_x}\hat{i} + a_{G_y}\hat{j} + ar_{A/G_y}\hat{j} - ar_{A/G_x}\hat{i} - w^2 r_{A/G_x}\hat{i} - w^2 r_{A/G_y}\hat{j}$$

$$i: \quad a_{A_x} = a_{G_x} - ar_{A/G_x} - w^2 r_{A/G_x}$$
$$j: \quad a_{A_y} = a_{G_y} + ar_{A/G_y} - w^2 r_{A/G_y} \quad (4,5)$$

Geometry and constraints

$$\bar{r}_{A/G} = 0\hat{i} + \frac{l}{2}\hat{j}$$

$$\begin{aligned} a_{Ax} &= -a_A \cos q \\ a_{Ay} &= -a_A \sin q \end{aligned} \qquad (6,7)$$

Other:

$$W = mg \qquad (8)$$

$$I_G = \frac{1}{12} m l^2 \qquad (9)$$

$$w = 0 \qquad (10)$$

Solving:

$$\boxed{a_A = \frac{4g \sin q}{\cos^2 q + 4\sin^2 q}}$$