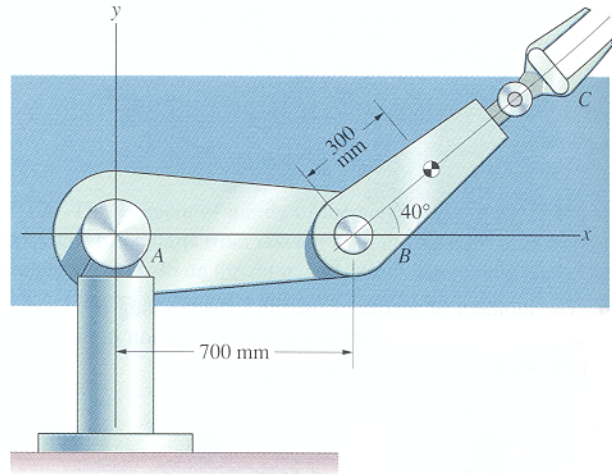


Example Problem - Le 22

7.27 Arm BC has a mass of 12 kg and the mass moment of inertia about its center of mass is 3 kg-m². If arm AB has a constant clockwise angular velocity of 2 rad/s and arm BC has a counterclockwise angular velocity of 2 rad/s and a clockwise angular acceleration of 4 rad/s²,

determine:

- a) the couple exerted on arm BC at B,
 - b) the reaction at B
- (taken from Dynamics, 2nd Edition by Bedford & Fowler)

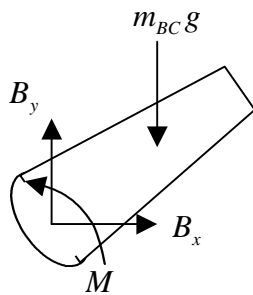


Strategy: Isolate system (arm BC)
 Kinetics (COLM, COAM RF)
 Kinematics (relative acceleration)

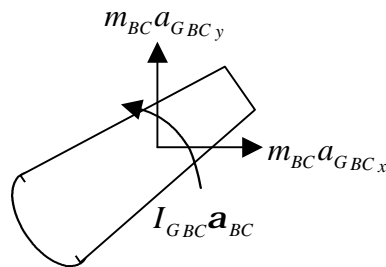
Given: $\bar{\omega}_{AB} = -2\hat{k}$, $\bar{\alpha}_{AB} = 0\hat{k}$, $\bar{\omega}_{BC} = 2\hat{k}$, $\bar{\alpha}_{BC} = -4\hat{k}$, $I_{GBC} = 3$, $m_{BC} = 12$

Kinetics:

FBD



KD



unk	eqs
M	(1)
B _x	(2)
B _y	(3)
a _{GBCx}	(4)
a _{GBCy}	(5)
a _{Bx}	(6)
a _{By}	(7)
a _{Ax}	(8)
a _{Ay}	(9)

COLM(RF)

x dir:

$$B_x = m_{BC} a_{GBCx} \tag{1}$$

y dir:

$$B_y - m_{BC} g = m_{BC} a_{GBCy} \tag{2}$$

COAM(RF) about B

$$M - m_{BC} g (0.3 \cos 40) = I_{GBC} \mathbf{a}_{BC} + m_{BC} a_{GBCy} (0.3 \cos 40) - m_{BC} a_{GBCx} (0.3 \sin 40) \tag{3}$$

Kinematics:

Arm BC:

$$\begin{aligned}\bar{a}_{GBC} &= \bar{a}_B + \bar{a}_{GBC/B} = \bar{a}_B + \bar{\mathbf{a}}_{BC} \times \bar{\mathbf{r}}_{GBC/B} - \mathbf{w}_{BC}^2 \bar{\mathbf{r}}_{GBC/B} \\ a_{GBC_x} \hat{i} + a_{GBC_y} \hat{j} &= a_{B_x} \hat{i} + a_{B_y} \hat{j} + \mathbf{a}_{BC} \hat{k} \times (r_{GBC/B_x} \hat{i} + r_{GBC/B_y} \hat{j}) - \mathbf{w}_{BC}^2 (r_{GBC/B_x} \hat{i} + r_{GBC/B_y} \hat{j}) \\ &= a_{B_x} \hat{i} + a_{B_y} \hat{j} + \mathbf{a}_{BC} r_{GBC/B_x} \hat{j} - \mathbf{a}_{BC} r_{GBC/B_y} \hat{i} - \mathbf{w}_{BC}^2 r_{GBC/B_x} \hat{i} - \mathbf{w}_{BC}^2 r_{GBC/B_y} \hat{j}\end{aligned}$$

$$\begin{aligned}i: \quad a_{GBC_x} &= a_{B_x} - \mathbf{a}_{BC} r_{GBC/B_y} - \mathbf{w}_{BC}^2 r_{GBC/B_x} \\ j: \quad a_{GBC_y} &= a_{B_y} + \mathbf{a}_{BC} r_{GBC/B_x} - \mathbf{w}_{BC}^2 r_{GBC/B_y}\end{aligned} \quad (4,5)$$

Arm AB:

$$\begin{aligned}\bar{a}_B &= \bar{a}_A + \bar{a}_{B/A} = \bar{a}_A + \bar{\mathbf{a}}_{AB} \times \bar{\mathbf{r}}_{B/A} - \mathbf{w}_{AB}^2 \bar{\mathbf{r}}_{B/A} \\ a_{B_x} \hat{i} + a_{B_y} \hat{j} &= a_{A_x} \hat{i} + a_{A_y} \hat{j} + \mathbf{a}_{AB} \hat{k} \times (r_{B/A_x} \hat{i} + r_{B/A_y} \hat{j}) - \mathbf{w}_{AB}^2 (r_{B/A_x} \hat{i} + r_{B/A_y} \hat{j}) \\ &= a_{A_x} \hat{i} + a_{A_y} \hat{j} + \mathbf{a}_{AB} r_{B/A_x} \hat{j} - \mathbf{a}_{AB} r_{B/A_y} \hat{i} - \mathbf{w}_{AB}^2 r_{B/A_x} \hat{i} - \mathbf{w}_{AB}^2 r_{B/A_y} \hat{j}\end{aligned}$$

$$\begin{aligned}i: \quad a_{B_x} &= a_{A_x} - \mathbf{a}_{AB} r_{B/A_y} - \mathbf{w}_{AB}^2 r_{B/A_x} \\ j: \quad a_{B_y} &= a_{A_y} + \mathbf{a}_{AB} r_{B/A_x} - \mathbf{w}_{AB}^2 r_{B/A_y}\end{aligned} \quad (6,7)$$

Geometry and Constraints:

$$\begin{aligned}\bar{\mathbf{r}}_{GBC/B} &= 0.3 \cos 40^\circ \hat{i} + 0.3 \sin 40^\circ \hat{j} \\ r_{B/A} &= 0.7 \hat{i} + 0.0 \hat{j} \\ \bar{\mathbf{a}}_A &= 0.0 \hat{i} + 0.0 \hat{j}\end{aligned} \quad (8,9)$$

Solving:

$\begin{aligned}B_x &= 35.4N \leftarrow \\ B_y &= 97.4N \uparrow \\ M &= 17.2Nm \text{CCW}\end{aligned}$
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