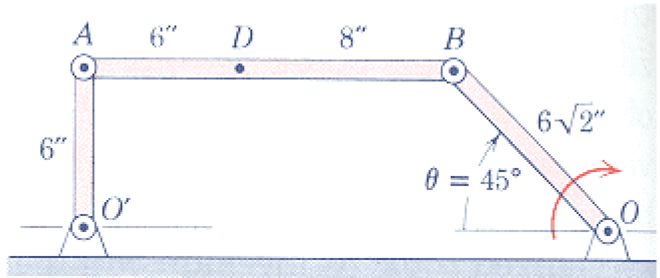
Example Problem - Le 13

Ex. Arm OB of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where θ=45°. Determine :
- (a) the velocity of point A,
- (b) the velocity of point D,
- (c) the angular velocity of link AB

(*taken from Engineering Mechanics, 3rd Edition by Meriam & Kraige*)



Vector Approach (Relative Motion)

Strategy:

1. Solve for $\bar{v}_B$ knowing $\bar{w}_{OB}$ and $\bar{r}_{B/O}$
2. Knowing $\bar{v}_B$ and $\bar{r}_{A/B}$, solve for $\bar{v}_A$ and $\bar{w}_{AB}$
3. Knowing $\bar{v}_B$ and $\bar{r}_{D/B}$, solve for $\bar{v}_D$

Part 1:

$$\bar{v}_B = \bar{v}_O + \bar{w}_{OB} \times \bar{r}_{B/O}$$

Since O is hinged and therefore the point of rotation, $\bar{v}_O = 0$. From the diagram, $\bar{w} = -10\hat{k}\ rad/s$ and $\bar{r}_{B/O} = -6\hat{i} + 6\hat{j}\ in$. Thus

$$\bar{v}_B = \left(-10\hat{k}\right) \times \left(-6\hat{i} + 6\hat{j}\right) = 60\hat{i} + 60\hat{j}\ in/s \quad (1)$$

Part 2:

$$\begin{aligned} \bar{v}_A &= \bar{v}_B + \bar{w}_{AB} \times \bar{r}_{A/B} \\ &= v_{B,x}\hat{i} + v_{B,y}\hat{j} + \left(w_{AB}\hat{k}\right) \times \left(r_{A/B,x}\hat{i} + r_{A/B,y}\hat{j}\right) \\ v_{A,x}\hat{i} + v_{A,y}\hat{j} &= v_{B,x}\hat{i} + v_{B,y}\hat{j} - w_{AB} r_{A/B,x}\hat{j} + w_{AB} r_{A/B,y}\hat{i} \end{aligned}$$

From the diagram, $v_{A,y} = 0$ and $\bar{r}_{A/B} = -14\hat{i} + 0\hat{j}\ in$. Thus we can write the last equation from above in component form:

$$\hat{i}: \quad v_{A,x} = v_{B,x} - w_{AB} r_{A/B,y}$$

$$v_{A,x} = v_{B,x} - 0 \quad (2)$$

$$\hat{j}: \quad v_{A,y} = v_{B,y} + w_{AB} r_{A/B,x}$$

$$0 = v_{B,y} + w_{AB}(-14) \quad (3)$$

Solving the two equations (2,3) for the two unknowns ($v_{A,x}$, w_{AB}):

$$v_{A,x} = 60\ in/s,\ v_{A,y} = 0\ in/s \quad \Rightarrow \quad \bar{v}_A = 60\hat{i}\ in/s$$

$$w_{AB} = 4.28 \quad \Rightarrow \quad \bar{w}_{AB} = 4.28\hat{k}\ rad/s$$

Part 3:

$$\bar{v}_D = \bar{v}_B + \bar{w}_{AB} \times \bar{r}_{D/B}$$
$$= v_{B,x}\hat{i} + v_{B,y}\hat{j} + \left(w_{AB}\hat{k}\right)\times\left(r_{D/B,x}\hat{i} + r_{D/B,y}\hat{j}\right)$$
$$v_{D,x}\hat{i} + v_{D,y}\hat{j} = v_{B,x}\hat{i} + v_{B,y}\hat{j} - w_{AB}r_{D/B,x}\hat{j} + w_{AB}r_{D/B,y}\hat{i}$$

From the diagram, $\bar{r}_{D/B} = -8\hat{i} + 0\hat{j}\ in$. Thus we can write the last equation from above in component form:

$$\hat{i}: \qquad v_{D,x} = v_{B,x} - w_{AB}r_{D/B,y}$$
$$v_{D,x} = v_{B,x} - 0 \qquad (4)$$
$$\hat{j}: \qquad v_{D,y} = v_{B,y} + w_{AB}r_{D/B,x}$$
$$v_{D,y} = v_{B,y} + 4.28(-8) \qquad (5)$$

Solving the two equations (4,5) for the two unknowns ($v_{D,x}$, $v_{D,y}$):

$$v_{D,x} = 60\,in/s\ ,\ v_{D,y} = 25.76\,in/s$$
$$v_D = 60\hat{i} + 25.76\hat{j}\ in/s$$

which is identical to the result obtained using the instantaneous center of velocity and the scalar approach.