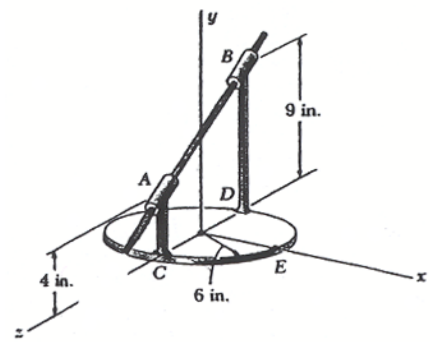
Example Problem - Le 10

15.20 A circular plate of 6 in. radius is supported by two bearings A and B as shown. The plate rotates about the road joining A and B with a constant angular velocity of 26 rad/s. Knowing that, at the instant considered, the velocity of point C is directed in the positive x direction, determine :

(a) the velocity of point E,
(b) the acceleration of points E

(*taken from an unknown source*)



Strategy: Pure kinematics
Vectors will be helpful

Kinematics

$$\bar{v}_E = \bar{w} \times \bar{r}_{E/O} \qquad (1)$$

$$\bar{a}_E = \bar{\alpha} \times \bar{r}_{E/O} + \bar{w} \times \left(\bar{w} \times \bar{r}_{E/O}\right) \qquad (2)$$

We need to come up with $\bar{w}$ and $\bar{r}_{E/O}$. Begin with the angular velocity - the plate is rotating about the axis defined by points A and B. From ConApps I (and hopefully Calc III), we can express the angular velocity vector as its magnitude and associated unit vector, hence

$$\bar{w} = w\,\hat{e}_w = w\,\hat{e}_{A/B} = w\,\frac{\bar{r}_{A/B}}{\left|\bar{r}_{A/B}\right|} \qquad (3)$$

Note that the appropriate rotation sense has been included with the position vector $\bar{r}_{A/B}$ via the right-hand rule. Make sure you understand this!

Obtaining the coordinates of A and B and forming the position vector:

$$A = (0, 4, 6)$$
$$B = (0, 9, -6)$$

$$\begin{aligned}\bar{r}_{A/B} &= \left(A_x - B_x\right)\hat{i} + \left(A_y - B_y\right)\hat{j} + \left(A_z - B_z\right)\hat{k} \\ &= (0-0)\hat{i} + (4-9)\hat{j} + (6+6)\hat{k} \\ &= 0\,\hat{i} - 5\,\hat{j} + 12\,\hat{k}\end{aligned}$$

$$\left|\bar{r}_{A/B}\right| = \sqrt{0^2 + 5^2 + 12^2} = 13$$

Substituting into (3)

$$\bar{w} = 26\,\frac{0\,\hat{i} - 5\,\hat{j} + 12\,\hat{k}}{13} = 0\,\hat{i} - 10\,\hat{j} + 24\,\hat{k} \quad {rad}/{s} \qquad (4)$$

Now we need the position vector from the point of rotation to the point of interest E. Since the plate is rotating about an axis, we can go from any point on the axis to E. Choose point A

$$\begin{aligned}\bar{r}_{E/A} &= \left(E_x - A_x\right)\hat{i} + \left(E_y - A_y\right)\hat{j} + \left(E_z - A_z\right)\hat{k} \\ &= 6\hat{i} - 4\hat{j} - 6\hat{k} \quad in\end{aligned} \qquad (5)$$

Substituting (4) and (5) into (1) and (2) and letting Maple do our cross-products for us:

$$\bar{v}_E = \bar{w} \times \bar{r}_{E/A} = 156\,\hat{i} + 144\,\hat{j} + 60\,\hat{k} \quad {in}/{s}$$

$$\bar{a}_E = \bar{\alpha} \times \bar{r}_{E/A} + \bar{w} \times \left(\bar{w} \times \bar{r}_{E/A}\right) = -4056\,\hat{i} + 3744\,\hat{j} + 1560\,\hat{k} \quad {in}/{s^2}$$