Problem CP-1 – Analysis of an Overhead Crane

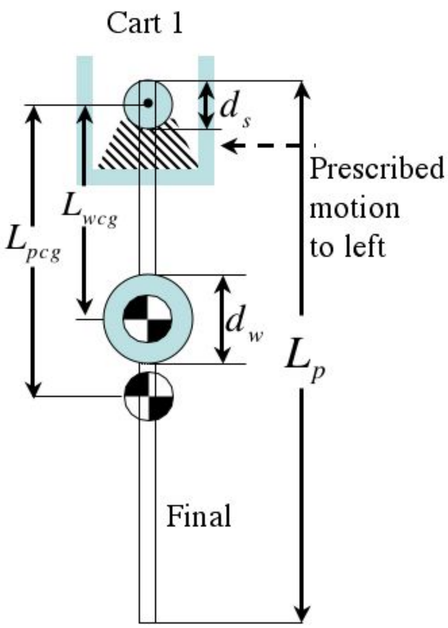

Figure 1: Crane system schematic.

The illustrated pendulum/cart system is used to model the motion of an overhead crane. If the cart is given a specified acceleration profile

$$b(t) = \frac{3597e^x(1 - e^x)}{(1 + e^x)^3} + \frac{3597e^y(1 - e^y)}{(1 + e^y)^3} \text{ cm/s}^2$$

where

$$x = -43.5668t + 9.38$$

$$y = -43.5668t + 31.1633$$

and $0 \text{ s} < t < 0.9 \text{ s}$, **determine the best location of the moveable weight's mass center L_{wcg} such that the pendulum's angular velocity is zero at $t = 0.9$ s**. Note that there will be an unknown force acting in the direction of the cart motion so that the cart experiences the specified acceleration.

Values for the various system parameters are provided in Table 1.

Table 1: Crane system parameters and their values.

Parameter	Value	Units
Pendulum mass, m_p	68.5	g
Moveable weight's mass, m_{add}	88	g
Pendulum length, L_p	43.2	cm
Sensor diameter, d_s	2.5	cm
Moveable weight's diameter, d_w	5	cm

Notes:

- Draw the system in a displaced orientation to obtain the governing differential equation. (This is the most important part of the analysis!).

- Write your resulting non-linear differential equation in the following format:

  $$[\text{something}]\ddot{\theta} + [\text{something else}]\sin(\theta) = [\text{something different}]b(t)\cos(\theta)$$

  When implementing this equation in Maple, it should look something like this:

diff_eq:=(IGp+IGw+mp*LGp^2+mw*Lw^2)*diff(theta(t),t$2)+(mp*LGp+mw*Lw)*g*sin(theta(t))= …
(right-hand side missing on purpose – we don't want to give you the whole answer!)

- Maple can numerically solve the non-linear differential equation of motion using the following syntax:

  ```
  > soln := dsolve({diff_eq, theta(0) = 0, D(theta)(0) = 0}, theta(t), numeric);
  ```

  To plot your solution, use the syntax

  ```
  > odeplot(soln, [t, D(theta)(t)], 0..2, numpoints = 300);
  ```

  Be sure to include > with(plots): at the beginning of your Maple worksheet. Once your Maple worksheet is working correctly for a particular value of L_{wcg}, you can then vary L_{wcg} to find the location where the angular velocity is zero at the end of the acceleration at $t = 0.9$ s.