

## Lab 4 — Numerical simulation of a crane

### Agenda

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<i>Time</i>	<i>Item</i>
10 min	Review agenda Introduce the crane problem
95 min	Lab activity

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I'll try to give you a 5-minute warning before the end of the lab period to put all the chairs back where they belong and to pack up your laptops.

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### Learning objectives:

By the end of the lab period, students should be able to:

- Implement a simulation diagram in Simulink and create an accompanying MATLAB m-file that solve the nonlinear equation of motion for a crane apparatus for two different move strategies.
- Compare the residual oscillation of an overhead crane's payload for single and double move strategies.

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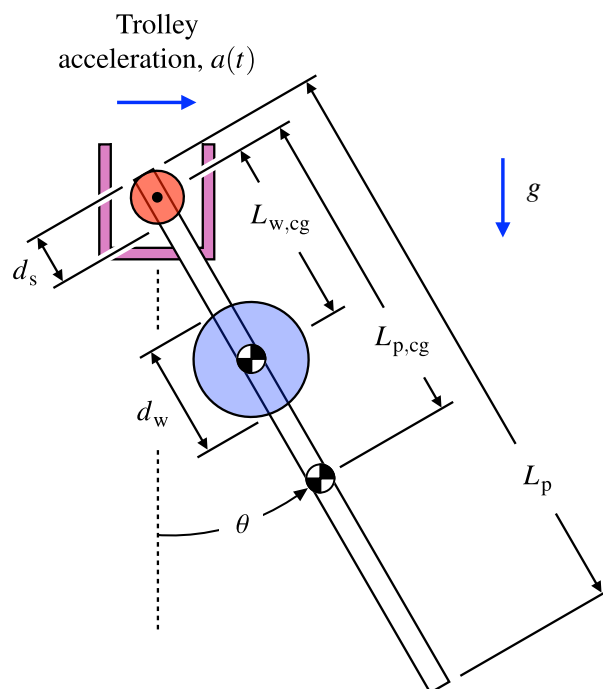
## Background

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The purpose of this lab is to create a numerical simulation of an overhead crane moving a suspended payload that you'll use to explore the system's behavior for two different motions of the crane trolley. In Lab 5, you'll *experimentally* explore the same system by collecting response data from a physical crane apparatus, and in a future homework assignment, you'll derive the system's governing equation of motion using the analytical tools presented in class. If your numerical simulation is well built and the experiment is well done, then you should find that both methods yield similar results.

### Modeling the crane

A schematic of the crane apparatus you'll be creating a numerical simulation for is illustrated in Figure 1. The system consists of four components: a cart (the "trolley") that is constrained to move horizontally with a prescribed acceleration  $a(t)$ ; a rod with mass  $m_p$  and length  $L_p$  (the "suspending cable") that is pivoted about a point near its top end and essentially acts like a pendulum; an attached circular weight of mass  $m_w$  and diameter  $d_w$  (the "payload") whose position along the rod is adjustable; and an attached sensor with diameter  $d_s$  that is mounted to the cart. The pivot axis passes through the center of the sensor and moves with the cart, and the sensor's mass is negligible compared to the rod and weight. The mass center locations for the rod and circular weight relative to the pivot point are denoted by  $L_{p,cg}$  and  $L_{w,cg}$ , respectively.



**Figure 1.** Schematic of a crane apparatus.

Applying first principles, the equation of motion describing the pendulum's angular displacement  $\theta$  measured counterclockwise from the downward position is

$$J\ddot{\theta} + k \sin \theta = -Ca(t) \cos \theta , \quad (1)$$

where the constants  $J$ ,  $k$ , and  $C$  are given by, respectively,

$$J = \frac{m_p L_p^2}{12} + m_p L_{p,cg}^2 + \frac{1}{2} m_w \left( \frac{d_w}{2} \right)^2 + m_w L_{w,cg}^2 , \quad (2)$$

$$k = g(m_p L_{p,cg} + m_w L_{w,cg}) , \quad (3)$$

$$C = m_p L_{p,cg} + m_w L_{w,cg} . \quad (4)$$

The distance to the rod's mass center from the pivot point is calculated as

$$L_{p,cg} = \frac{L_p - d_s}{2} . \quad (5)$$

Values for the various system parameters are provided in Table 1. Note that because the placement of the circular weight along the rod can be adjusted, a numerical simulation of (1) will yield different results for different values of  $L_{w,cg}$ .

**Table 1.** Parameter values for the crane apparatus.

Parameter	Value with units
$m_p$	68.5 g
$m_w$	88 g
$L_p$	43.2 cm
$d_w$	5 cm
$d_s$	2.5 cm

### Strategies for moving the trolley

Precise payload positioning by an overhead crane (especially when performed by an operator using only visual feedback to position the payload) is difficult because the payload can exhibit a pendulum-like swinging motion. In this lab, you'll see that there are strategies for moving the payload that reduce the swinging motion that occurs after the crane has stopped moving. Our basis for comparison is the maximum angular velocity the rod (i.e., the pendulum) experiences during its residual swing after the trolley has come to a stop.

The simplest approach to moving the crane to its destination is to move it in a single motion. For this lab, a **single move strategy** for the crane is executed by accelerating the trolley according to

$$a(t) = \frac{7194 e^x (e^x - 1)}{(1 + e^x)^3} \text{ cm/s}^2 , \quad (6)$$

where the exponent  $x$  is given by

$$x = -43.5668 t + 9.38 . \quad (7)$$

An alternative approach is to move the trolley in a sequence of two moves. This **double move strategy** is a well known strategy for moving an overhead crane so that the payload arrives at the desired position with very little residual oscillation. For this lab, the corresponding trolley acceleration is

$$a(t) = \frac{3597 e^x (e^x - 1)}{(1 + e^x)^3} + \frac{3597 e^y (e^y - 1)}{(1 + e^y)^3} \text{ cm/s}^2 , \quad (8)$$

where  $x$  is the same as in (7) and the exponent  $y$  is given by

$$y = -43.5668 t + 31.1633 . \quad (9)$$

For both move strategies, the trolley essentially comes to a stop after about 1 s.

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## Lab activity

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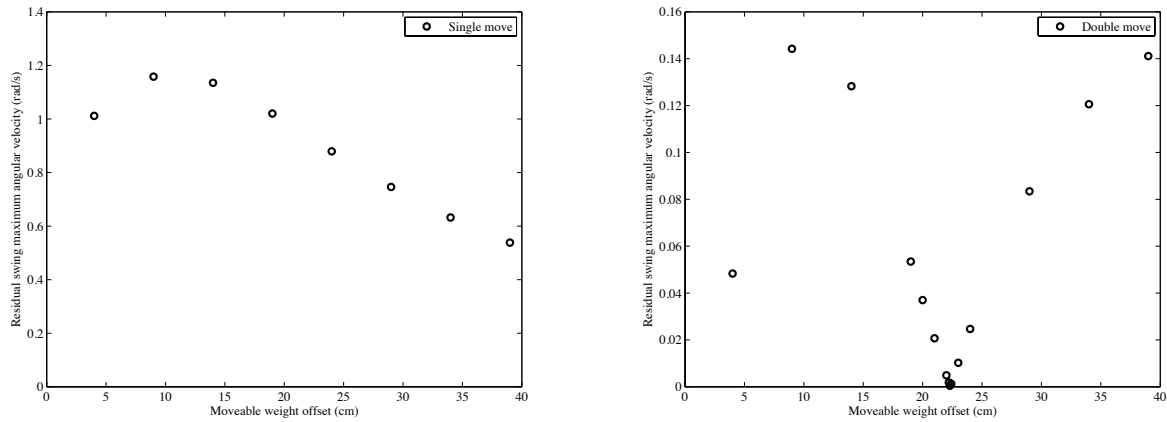
Create a simulation diagram in Simulink and an accompanying MATLAB m-file that solve the equation of motion (1) for both move strategies:

- Have your numerical simulation return the pendulum's angular velocity  $\dot{\theta}$  over time  $t$  when subjected to the cart acceleration  $a(t)$ , starting at rest and initially in the downward position:  $\theta(0) = 0$  and  $\dot{\theta}(0) = 0$ . Be sure to run your simulation for longer than 1 s. A simulation time of 3 s should work.
- After about 1 s into the simulation, the trolley essentially comes to a stop, but the pendulum continues to swing. For both move strategies, use your simulation to explore how the maximum angular velocity of the pendulum during this residual swing varies when you place the circular weight at the locations listed in Table W1 in the Lab 4 worksheet. Record your simulation results in the appropriate sections of Table W1.

One way to find the maximum angular velocity is to plot the pendulum's angular velocity over time and then use the data cursor in the figure window to read off the angular velocity at a peak after about 1 s.

- **For the double move strategy only**, expand the data set in Table W1 using trial and error to determine the circular weight's position,  $L_{opt}$ , to within 0.1 cm that minimizes the pendulum's maximum angular velocity during its residual swing. Use the blank space in Table W1 to record the position locations that you tried and the corresponding simulation results. *Highlight or circle the row in Table W1 that corresponds to the position that minimizes the residual swing maximum angular velocity.*
- For the single move strategy, create a plot in MATLAB that shows how the maximum angular velocity of the pendulum's residual swing varies with the circular weight's location *in centimeters* along the rod (that is, you should have the angular velocity on the vertical axis and weight location in centimeters on the horizontal axis).
- Make a similar plot for the double move strategy. *Be sure to use all of the results you collected.*
- Use markers and don't connect them with a line. Also, be sure to label your axes and include units. Include your initials and the date in the title of each figure, and remove the gray border around the figures.

Your plots of how the pendulum's maximum angular velocity during its residual swing changes with the placement of the circular weight along the rod for the single and double move strategies should look similar to the plots shown in Figure 2.



**Figure 2.** Simulation results for the pendulum’s maximum angular velocity during its residual swing with the circular weight placed at several locations along the rod for single and double move strategies.

***Deliverables:***

You will turn in the following items along with Lab 5 on Friday of Week 10:

1. A completed Lab 4 worksheet with numerical simulation results (Table W1).
2. A printout of your Simulink model used for simulation.
3. A printout of your MATLAB m-file used to run your simulation.