

## Lab 2 — Numerical simulation of a swinging pendulum

### Agenda

<i>Time</i>	<i>Item</i>
10 min	Review agenda Introduce the swinging pendulum problem
95 min	Lab activity

I'll try to give you a 5-minute warning before the end of the lab period to put all the chairs back where they belong and to pack up your laptops.

### Learning objectives:

By the end of the lab period, students should be able to:

- Implement a simulation diagram in Simulink and create an accompanying MATLAB m-file that solve the nonlinear equation of motion for a pendulum apparatus.
- Modify their Simulink model to conditionally stop a simulation.
- Input tabulated simulation results as data vectors in MATLAB and plot these results.

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## Background

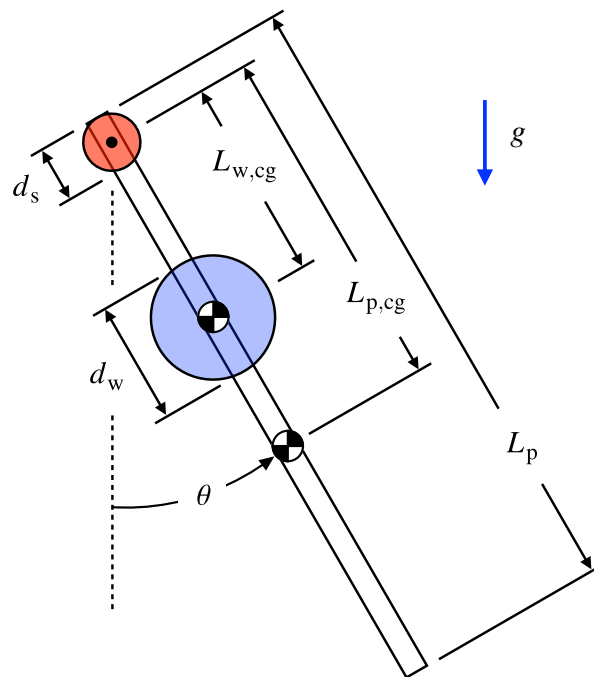
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The purpose of this lab is to create a numerical simulation of a swinging pendulum that you'll use to explore the system's behavior. You'll find that it's pretty difficult to intuitively predict this behavior. By exploring the system's response in a numerical simulation, you'll start to gain an understanding of the complexity of a rigid body in rotation.

In Lab 3, you'll *experimentally* explore the same system by collecting response data from a physical pendulum. In a future homework assignment, you'll predict the pendulum's behavior through *analytical* means using the material presented in class. If your numerical simulation is well built, the experiment is well done, and the theoretical analysis is well justified, you should find that all three methods yield similar results.

### Modeling the pendulum

A schematic of the pendulum apparatus you'll be creating a numerical simulation for is illustrated in Figure 1. The system consists of three components: a rod with mass  $m_p$  and length  $L_p$  that is pivoted about a point near its top end; an attached circular weight of mass  $m_w$  and diameter  $d_w$  whose position along the rod is adjustable; and an attached sensor with diameter  $d_s$ . The fixed pivot axis passes through the center of the sensor, and the sensor's mass is negligible compared to the rod and weight. The mass center locations for the rod and circular weight relative to the pivot point are denoted by  $L_{p,cg}$  and  $L_{w,cg}$ , respectively.



**Figure 1.** Schematic of a pendulum apparatus.

Applying first principles, the equation of motion describing the pendulum's angular displacement  $\theta$  measured counterclockwise from the downward position is

$$J\ddot{\theta} + k \sin \theta = 0 , \quad (1)$$

where the constants  $J$  and  $k$  are given by, respectively,

$$J = \frac{m_p L_p^2}{12} + m_p L_{p,cg}^2 + \frac{1}{2} m_w \left( \frac{d_w}{2} \right)^2 + m_w L_{w,cg}^2 , \quad (2)$$

$$k = g(m_p L_{p,cg} + m_w L_{w,cg}) . \quad (3)$$

The distance to the rod's mass center from the pivot point is calculated as

$$L_{p,cg} = \frac{L_p - d_s}{2} . \quad (4)$$

Values for the various system parameters are provided in Table 1.

**Table 1.** Parameter values for the pendulum apparatus.

Parameter	Value with units
$m_p$	68.5 g
$m_w$	88 g
$L_p$	43.2 cm
$d_w$	5 cm
$d_s$	2.5 cm

Note that because the placement of the circular weight along the rod can be adjusted, a numerical simulation of (1) will yield different results for different values of  $L_{w,cg}$ .

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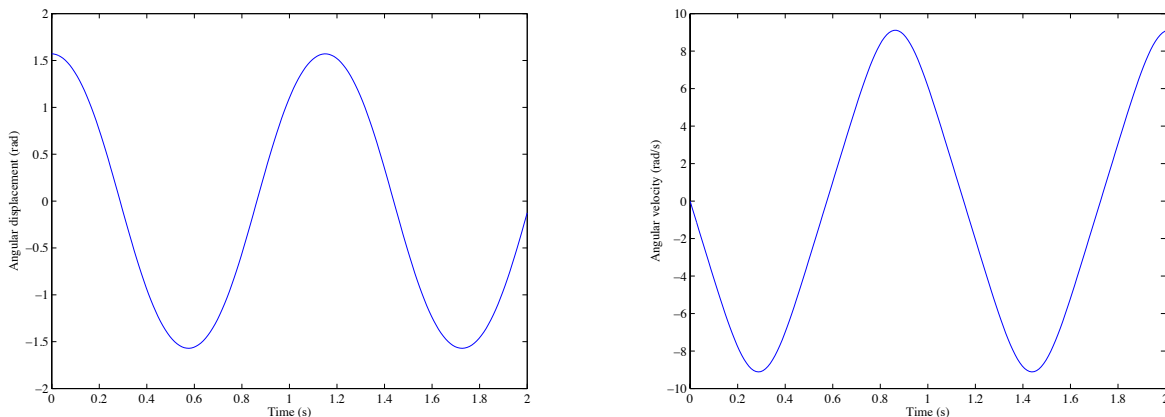
## Lab activity

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### Part A: Building a basic simulation diagram

Create a simulation diagram in Simulink and an accompanying MATLAB m-file that solve the equation of motion (1):

- Have your numerical simulation return the pendulum's angular displacement  $\theta$  and its angular velocity  $\dot{\theta}$  over time  $t$  when released from rest and initially in the horizontal position:  $\theta(0) = \frac{\pi}{2}$  rad and  $\dot{\theta}(0) = 0$ .
- Simulate the pendulum's response for 2 s with the circular weight placed at  $L_{w,cg} = 20$  cm.
- Plot the angular displacement and velocity over time, and confirm that you obtain the same plots as shown in Figure 2.



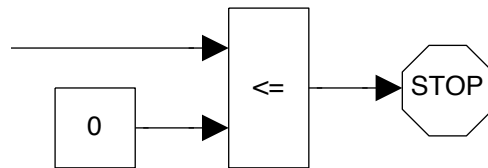
**Figure 2.** Simulation results for the pendulum released from rest in the horizontal position with the circular weight placed at 20 cm from the pivot.

### Part B: Making the simulation stop at the bottom of the swing

Let's investigate how the pendulum's angular velocity at the bottom of its swing changes as we vary the circular weight's position along the rod. Let's also see how the time it takes to get to the bottom of the swing varies with the weight's location. To do this easily, we'll want to have the simulation stop once the pendulum reaches the vertical position. This way, to find the angular velocity and time instant we're interested in, all we need to do is take a look at the last element in the angular velocity and time vectors, respectively.

To have the simulation stop, we need to include a “Stop Simulation” block in the simulation diagram, which you can find under “Sinks” in the Library Browser. The “Stop Simulation” block should activate only when the pendulum swings just past the vertical position, or when  $\theta \leq 0$ . We can specify this command using the “Relational Operator” block under “Logic and Bit Operations.” This block requires two inputs, where the first input is compared to the second using a specified logical operator.

Modify your simulation diagram from Part A so that the simulation will stop when the pendulum reaches the bottom of its swing. The section of your Simulink model containing the “Stop Simulation” and “Relational Operator” blocks should look similar to the diagram in Figure 3.



**Figure 3.** The simulation stops when the first input signal becomes less than or equal to zero.

Now, run your simulation again and check that it does indeed stop at the bottom of the swing by making sure that the last element of your angular displacement vector is zero, or very close to it. If you called the angular displacement vector `theta`, then the code

```
theta(end)
```

will return the last entry of `theta`.

### Part C: Exploring how the swing characteristics change with weight location

Use your simulation to explore how the angular velocity magnitude at the bottom of the swing and how long it takes to get to this position changes when you place the circular weight at different locations along the rod:

- Run your simulation using each of the locations listed in Table W1 in the Lab 2 worksheet. Record your simulation results in the appropriate sections of Table W1.
- Expand the data set in Table W1 using trial and error to determine the circular weight’s position  $L_{\max}$  to within 0.1 cm that maximizes the pendulum’s angular velocity magnitude at the bottom of the swing. Use the blank space in Table W1 to record the position locations that you tried and the corresponding simulation results. *Highlight or circle the row in Table W1 that corresponds to the maximum angular velocity magnitude.*

- Using all of the results you collected, create a plot in MATLAB that shows how the pendulum's angular velocity magnitude at the bottom of the swing varies with the circular weight's location *in centimeters* along the rod (that is, you should have angular velocity magnitude on the vertical axis and weight location in centimeters on the horizontal axis).
- Make a similar plot for the swing time (i.e., plot the swing time on the vertical axis and weight location in centimeters on the horizontal axis).
- Use markers and don't connect them with a line. Also, be sure to label your axes and include units. Include your initials and the date in the title of each figure.

To make these plots, you'll first need to put the simulation results listed in Table W1 into MATLAB as vectors. For example, let's say I want to transfer the sample results tabulated in Table 2 to MATLAB. (These results have nothing to do with your lab assignment – they're just being used as an example.)

**Table 2.** Results from vibration testing of a steel bracket.

Mode number	Frequency of vibration (Hz)	Damping
1	80	0.102
2	295	0.020
3	435	0.018
4	463	0.018

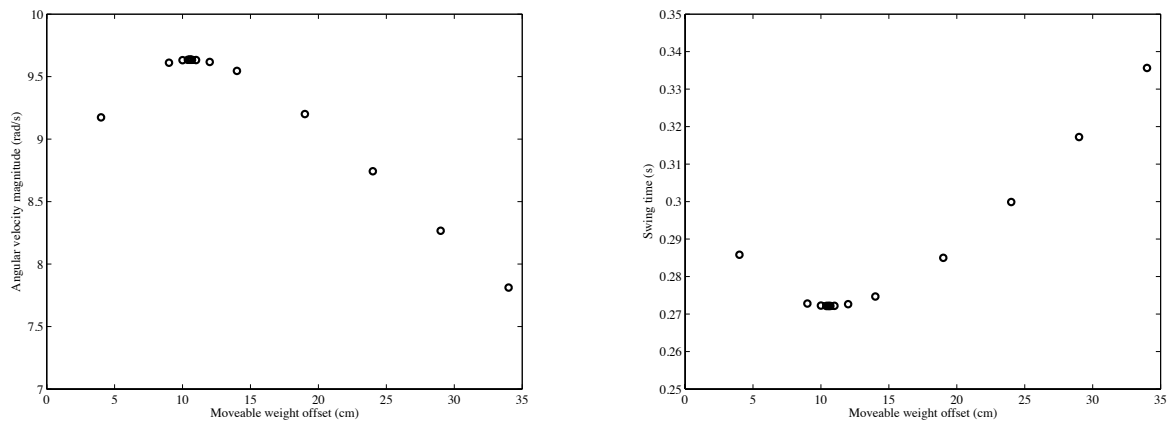
In my MATLAB m-file, I would include the lines of code

```
mode = [1; 2; 3; 4];
freq = [80; 295; 435; 463];
damp = [0.102; 0.020; 0.018; 0.018];
```

to create the vectors `mode`, `freq`, and `damp` for the mode number, vibration frequency, and damping data, respectively. Now that the data is stored in MATLAB, I can then plot, say, the frequency of vibration against the mode number with circles for markers:

```
plot(mode, freq, 'o')
```

Your plots of how the pendulum's angular velocity magnitude at the bottom of its swing and the corresponding swing time change with the placement of the circular weight along the rod should look similar to the plots shown in Figure 4.



**Figure 4.** Simulation results for the pendulum's angular velocity magnitude at the bottom of its swing and the corresponding swing time when released from rest in the horizontal position with the circular weight placed at several locations along the rod.

***Deliverables:***

For this lab activity, you need to turn in the provided worksheet with a completed section of numerical results (Table W1) and all requested plots and documentation attached.