Conservation and Accounting Review

by P. Cornwell

In ES201 the basic idea of conservation and accounting was investigated and you were introduced to conservation of mass, charge, linear momentum, angular momentum, and energy and an accounting equation for entropy. In ES204 we will investigate the application of these principles to mechanical systems. In this class we will be primarily using conservation of linear momentum, conservation of angular momentum and conservation of energy. The rate form of these equations are shown below.

Linear Momentum:
$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F} + \sum_{in} \dot{m}_i \vec{v}_i - \sum_{out} \dot{m}_o \vec{v}_o \tag{1}$$

where for a system consisting of a single object $\vec{P}_{sys} = m\vec{v}_G$ or for a system of several objects $\vec{P}_{sys} = \sum_{i=1}^{n} (m\vec{v}_G)_i$

Angular Momentum about point O:
$$\frac{d\vec{L}_{sys_0}}{dt} = \sum \vec{M}_o + \sum_{in} \vec{r} \times \dot{m}_i \vec{v}_i - \sum_{out} \vec{r} \times \dot{m}_o \vec{v}_o \tag{2}$$

where for the plane motion of a single rigid body $\vec{L}_{sys_0} = I_G \vec{\omega} + \vec{r} \times m\vec{v}_G$

Energy:
$$\frac{dE_{sys}}{dt} = \dot{Q} + \dot{W} + \sum_{in} \dot{m}_i \left(h + \frac{v^2}{2} + gz \right)_i - \sum_{out} \dot{m}_o \left(h + \frac{v^2}{2} + gz \right)_o \tag{3}$$

where $E_{sys} = E_k + E_G + E_s + U$

In this class we will primarily be looking at closed systems so Eqs (1-3) become:

Linear Momentum:
$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F} \tag{4}$$

Angular Momentum:
$$\frac{d\vec{L}_{sys_0}}{dt} = \sum \vec{M}_o \tag{5}$$

Energy:
$$\frac{dE_{sys}}{dt} = \dot{W} \tag{6}$$
$$\Delta E_{sys} = W$$

In most dynamics books the use of Eqs. (4-5) are referred to as direct applications of Newton's 2nd Law (Chapters 12 and 16 in Beer and Johnson), the use of the finite time forms of Eqs. (4-5) are referred to as impulse momentum methods (Chapters 13 and 17 in Beer and Johnson) and the use of the finite time form of Eq. (6) is referred to as the Work-Energy method (Chapters 13 and 17 in Beer and Johnson).

A summary of the equations you are expected to know from ES201 and how they are typically referred to in the text is shown below.

Principle	ES201 Name	Dynamics Name	Comments
$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F}$$ $$\frac{d\vec{L}_{sys_0}}{dt} = \sum \vec{M}_o$$	Rate form for conservation of linear and angular momentum for a closed system.	Direct application of Newton's Laws	When to use: - want to find forces and/or accelerations - want to find velocities and/or distance traveled (which can be found by separating variables and integrating the basic kinematic relationships) Other: - Be careful! These are vector equations..
$$\Delta \vec{P}_{sys} = \int_{t_1}^{t_2} \vec{F}dt$$ $$\Delta \vec{L}_{sys_0} = \int_{t_1}^{t_2} \vec{M}_0 dt$$ or if there are impulsive loads acting on the system $$\Delta \vec{P}_{sys} = \sum \vec{F}_i \Delta t$$ $$\Delta \vec{L}_{sys_0} = \sum (\vec{M}_0)_i \Delta t$$ where F_i and M_i are only external impulsive forces and moments acting on the system.	Finite time form of conservation of linear and angular momentum for a closed system.	Impulse-momentum methods	When to use: - have an impact or impulsive forces - the system consists of several objects - given a force as a function of time - want to find velocities, times, or forces (especially impulsive forces) Other: - Be careful! These are vector equations..
$$\Delta E_{sys} = W$$	Finite time form of conservation of energy for an adiabatic closed system.	Work-energy methods.	When to use: - have two locations in space - given a force as a function of position - want to find velocities, distances, or forces (sometimes) Other: - This is a scalar equation