Appendix A - Mass Moments of Inertia

In this appendix I will tell you all you need to know about mass moments of inertia (at least for this class).

A.1 What is the mass moment of inertia?

The mass moment of inertia is a measure of an objects resistance to rotation, similar to mass being a measure of an objects resistance to translation. It depends on the distribution of mass of an object and the axis about which the mass moment of inertia is to be calculated. The mass moment of inertia is defined to be:

$$I = \int r^2 dm$$

The mass moments of inertia for some common shapes are shown on the following page.

Parallel Axis Theorem

Given the mass moment of inertia about an axis passing through the center of gravity, the mass moment of inertia about an axis passing through any other point may be determined using the parallel axis theorem as shown below:

$$I_O = I_G + mr^2$$

where
  r = the distance between G and O
  I_O = mass moment of inertia about an axis passing though O
  I_G = mass moment of inertial about the center of gravity.

Radius of Gyration

The radius of the gyration is the quantity that when squared and multiplied by the mass you get the mass moment of inertia. That is
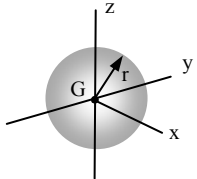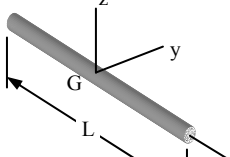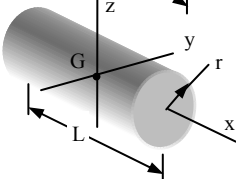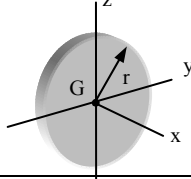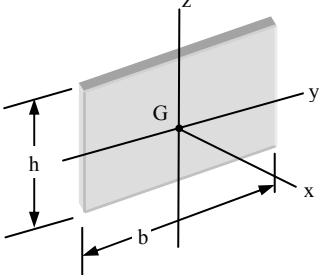
$$I_O = mk_O^2$$

where
  k_O = the radius of gyration about point O
  m = mass of the object
  I_O = mass moment of inertia of the object about an axis passing through O.

It is basically just an alternative way of giving you the mass moment of inertia.

Mass moment of Inertia of some Common Shapes

Solid Sphere $I_x = I_y = I_z = \dfrac{2}{5} mr^2$	
Slender Rod $I_y = I_z = \dfrac{1}{12} mL^2$	
Solid Circular Cylinder $I_x = \dfrac{1}{2} mr^2$ $I_y = I_z = \dfrac{1}{12} m\left(L^2 + 3r^2\right)$	
Thin Disk $I_x = \dfrac{1}{2} mr^2$ $I_y = I_z = \dfrac{1}{4} mr^2$	
Thin Rectangular Plate $I_x = \dfrac{1}{12} m\left(b^2 + h^2\right)$ $I_y = \dfrac{1}{12} mh^2$ $I_z = \dfrac{1}{12} mb^2$	
Brick $I_x = \dfrac{1}{12} m\left(b^2 + h^2\right)$ $I_y = \dfrac{1}{12} m\left(h^2 + d^2\right)$ $I_z = \dfrac{1}{12} m\left(b^2 + d^2\right)$	