

# Mathematics of Image Processing

## Worksheet #6 - The Haar wavelet filter bank

Name: \_\_\_\_\_

Box #: \_\_\_\_\_

1. Let  $l = \frac{1}{2}[1, 1]$ ,  $h = \frac{1}{2}[1, -1]$ . Let  $X = [x_0, x_1, \dots, x_7]^t$  be an eight point sample. The single stage filter bank determined by  $l$  and  $h$  may be implemented computationally by matrix multiplication:

$$X_1 = W_1 X.$$

Compute the matrix  $W_1$ .

2. Now let  $X_2$  be the vector obtained by doing a second stage of a filter bank on the lowpass output of the first stage and keeping the high pass output the same. Compute the matrices  $V_2$  and  $W_2$  satisfying:

$$X_2 = V_2 X_1,$$

$$X_2 = W_2 X.$$

3. Compute  $V_3$  and  $W_3$  for the third stage. Make sure you draw a diagram.

4. Verify that the pure wave forms defined by the rows of  $W_3$  are orthogonal. Draw a bar graph representing the pure (analysis) wave forms.

5. How can you modify  $l$  and  $h$  to obtain orthonormal pure (analysis) wave forms.