MA 439 - Math of Image Processing
Assignment #5

Professor Broughton

Due Date: Tuesday October 15

Name:_____ Box #:_____

1. Let X denote a monochrome $m \times n$ image and let M be the maximum value of $\left|\widehat{X}(k,l)\right|$ $0 \le k < m$ and $0 \le l < n$. Define X_c and $\widehat{X_c}$ by:

$$\begin{aligned}
\widehat{X_c}(k,l) &= 0, \text{ if } \widehat{X_c}(k,l) < \frac{c}{100}M, \\
\widehat{X_c}(k,l) &= \widehat{X}(k,l), \text{ otherwise} \\
X_c &= \mathcal{F}^{-1}(\widehat{X_c}).
\end{aligned}$$

   Observe that $X_0 = X$ and that $X_{100+\varepsilon} = 0$, for positive ε. Pick an image X and construct the following table for a variety of values of c, measured in percent, for at least at least 10 values. Some sample values c that you might try are shown. The quantities PRC and D are the percentage of retained coefficients and distortion expressed as percents:

$$\begin{aligned}
PRC &= 100\frac{mn - \#\left\{(k,l) : \widehat{X_c}(k,l) = 0\right\}}{mn} \\
D &= 100\frac{\|X - X_c\|^2}{\|X\|^2}
\end{aligned}$$

c	PRC	PD
.01		
.02		
.1		
.2		

   Note that the second Matlab assignment has an example showing how to form X_c.

2. Construct a rate distortion curve by plotting PRC against PD.

3. For three different images, find the largest c so that X_c looks like a natural image. Can you devise a rule of thumb for a maximal acceptable percentage distortion.

4. Progressive resolution. Pick any image. Show five progressively finer resolutions of the image as follows.

   - Pick an X.
   - Construct $\widehat{X}$.
   - Construct $\widehat{X}_n$ by selecting a set of coefficients in the center of the shifted DFT. As n get larger the set of coefficients gets larger, and $\widehat{X}_5 = \widehat{X}$
   - construct and show $X_n = \mathcal{F}^{-1}\widehat{X}_n$.

   Send your Matlab script (question 4 only) electronically.