MA311- Probability - Test #1

Professor Broughton — March 30, 1998

Name:____________________ Box #____________________

Instructions

- Answer all the questions directly on the test.
- Show all the necessary work and write your answers out neatly in English sentences. Use mathematical notation to express your answers, not *Maple* notation
- It is not necessary to use your computer to answer all of the questions but you can use it to obtain graphs, evaluate functions, solve equations, etc. If you use *Maple* be sure to say so by some sentence such as: Using *Maple* the above integral equals ....
- Recall that you may use notes that you can fit on one standard sheet of paper. On your computer you may start off with one blank *Maple* worksheet only. Please hand in your sheet of notes with your test.

Question	Points
1	
2	
3	
4	
Total	

1 Quick and dirty numerical calculations:

1.a Three standard, fair dice are thrown. What is the probability of getting a die sum of 5.

1.b A couple has children until a child of each sex has been born. What is the probability that there are at most 4 children.

2 The three events A, B, C are independent, with probabilities $P(A) = a$, $P(B) = b$ and $P(C) = c$. Suppose that E is the event consisting of outcomes that belong to exactly one of A, B or C. Determine $P(E)$ as a polynomial in a, b, c. In your justification show the appropriate Venn diagram.

3 An urn contains r red marbles and g green marbles. Two marbles are selected in order and their colours noted. Let A_C denote the event that the first marble has colour $C \in \{R, G\}$ and B_C the corresponding event for the second marble. We are going to compare conditional probabilities, under the two schemes of replacement and non replacement of the first marble.

3.a First, fill in the missing items in the following table (I put some in here to help you answer the question a little faster):

	$P(A_R)$	$P(A_G)$	$P(B_R \vert A_R)$	$P(B_G \vert A_R)$	$P(B_R \vert A_G)$	$P(B_G \vert A_G)$
replace			$\frac{r}{r+g}$	$\frac{g}{r+g}$	$\frac{r}{r+g}$	$\frac{g}{r+g}$
don't replace	$\frac{r}{r+g}$	$\frac{g}{r+g}$				

3.b If replacement is used then $P(B_R) = \frac{r}{r+g}$. Is this the same as when there is no replacement? Answer the question by computing $P(B_R)$ without replacement, using the information in part 1.a. above.

3.c Which scheme makes A_G and B_R independent.

4 A system has 3 components of type A, namely, $\{A_1, A_2, A_3\}$, and one component of type B. The components of type A are either in state a (available) or state n, (not available). Component B is one of three mutually exclusive states: d (dead), s (sluggish) or f (fast). The components of type A have identical characteristics, and the system is constructed so that a component's performance is independent of all the other components. The availability probabilities are given by:

$$P(A_i = a) = u,\ P(A_i = n) = v,\ P(B = d) = x,\ P(B = s) = y,\ P(B = f) = z.$$

The system will be considered available if:

- all three of the A_i available, or
- B is not dead and two A_i are available, or
- B is fast and at least one A_i is available.

Let E be the event that the system is available

4.a Find a sample space for the system.

4.b Compute $R = P(E)$ the availability of the system as a polynomial in u, v, x, y, z.

4.c Suppose that the three states for B are equally likely and that the following conditional probabilities have been determined:

$$P(E|B = d) = \frac{1}{8},\ P(E|B = s) = \frac{3}{8},\ P(E|B = f) = \frac{7}{8}$$

Then compute the probability that B is fast if the system is available.