Disco I - WorkSheet 11

Professor Broughton

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1. Fibonacci and catalan numbers

In the next couple of worksheets we are going determine formulas for the Fibonacci and Catalan numbers by using generating functions. In this worksheet we will come up with a couple of examples of how the Fibonacci and Catalan numbers arise and also derive the recursion formulas.

1.1. Fibonacci Numbers

1. Let B_n be the set of all sequences of bits of length n in which two consecutive zeros never occurs. For example $01101 \in B_5$, but $11001 \notin B_5$. Find the first 5 sets. In finding these sets organize your work so that you can derive B_3 from B_2, B_4 from B_3 and so on.

2. Define $b_n = |B_n|$. Find a recursion relation for the sequence $\{b_n\}$.

3. Let A_n be the set of subsets of $\{1, 2, 3, \ldots, n\}$ in which no two consecutive integers occur. For example $\{1, 3, 5\} \in A_5$; but $\{1, 3, 4\} \notin A_5$. Now repeat question 1 with this set up.

4. Let $a_n = |A_n|$. Find a recursion relation for the sequence $\{a_n\}$. What is the relation between a_n and b_n?

1.2. Catalan numbers

5. (See page 145) Let C_n be the set of all bracketed expressions formed from $x_0 \star x_1 \star x_2 \star \cdots x_n$. For example

$$C_1 = \{x_0 \star x_1\};\ C_2 = \{(x_0 \star x_1) \star x_2;\ x_0 \star (x_1 \star x_2)\}.$$

   Find C_3. Then find C_4 by building on your construction for C_3.

6. Let $c_n = |C_n|$. Find a recursion relation for the sequence $\{c_n\}$.

7. Let P_n be the set of all triangulated regular polygons with $n + 2$ sides. For example P_1 and P_2 are given below. Find P_3. Then find P_4 by building on your construction for P_3: A suggestion for building is given below.

8. Let $p_n = |P_n|$: Find a recursion relation for the sequence $\{p_n\}$: what is the relation between p_n and c_n?

   (picures only on the class handout)