Disco I- answers to worksheet 4

1. Associativity of products

1.a Let ® = (1; 2; 3)(4; 5); ¯ = (2; 3)(4; 5); ° = (1; 5): Compute (®¯)° and ®(¯°): What do you observe?

(®¯)° = [(1; 2; 3)(4; 5)(2; 3)(4; 5)](1; 5) = (1; 3)(1; 5) = (1; 3; 5)
®(¯°) = (1; 2; 3)(4; 5)[(2; 3)(4; 5)(1; 5)] = (1; 2; 3)(4; 5)(1; 5; 4)(2; 3) = (1; 3; 5)

1.b Compute 3(®¯)° = $3^{(\text{®¯})\text{°}}$ and 3®(¯°) = $3^{\text{®}(\text{¯°})}$ step by step. What do you observe?

3(®¯)° = 1° = 5
3®(¯°) = 1(¯°) = 5

2. Let ± = (3; 4): Write down all the association schemes for ®¯°± and verify that two of them are equal.

®(¯(°±)); ®((¯°)±)); (®¯)(°±); ((®¯)°)±; (®(¯°))±

2. Commutativity of Products

3.a Let ® = (1; 2; 3; 4; 5); ¯ = (3; 5; 6). Does ®¯ = ¯®?

®¯ = (1; 2; 5)(3; 4; 6)
¯® = (1; 2; 3)(4; 5; 6)

They do not commute.

3.b Next try to see if ° = (1; 3; 5); ± = (2; 4; 6) commute. They do commute.

3.c Write down a conjecture on commutativity of cycles. O¤er at least 3 examples as evidence. Cycles which are disjoint commute.

3. Powers

4 Let ® = (1; 2; 3); ¯ = (6; 7) and ° = ®¯: Compute the powers ®^{n}; ¯^{n}; °^{n} in a table format until you see a pattern emerge. What is the pattern?

n	®^{n}	¯^{n}	°^{n}
1	(1; 2; 3)	(6; 7)	(1; 2; 3)(6; 7)
2	(1; 3; 2)	id	(1; 3; 2)
3	id	(6; 7)	(6; 7)
4	(1; 2; 3)	(id	(1; 2; 3)
5	(1; 3; 2)	(6; 7)	(1; 3; 2)(6; 7)
6	id	id	id

The pattern will repeat itself every 3'rd row in the …rst column, every second row in the second column, and every sixth row in the third.

5. Make a prediction if ® = (1; 2; 3; 4; 5); ¯ = (7; 8; 9) and ° = ®¯: The elements ®; ¯ and ° = ®¯ have orders 5, 3 and 15 respectively.