

Disco I - answers to worksheet #2

1. The symmetric group

- Write out all possible “shu - es” or permutations of S_3 the symmetric group on three symbols. Write down the list notation and the disjoint cycle notation for each.

list	cycle
1; 2; 3	id
1; 3; 2	(2; 3)
2; 1; 3	(1; 2)
2; 3; 1	(1; 3; 2)
3; 1; 2	(1; 2; 3)
3; 2; 1	(1; 3)

- Based on number 1, determine the number of 4-shu - es and 5- shu - es, i.e., ...nd the cardinalities of S_4 and S_5 :

$$|S_4| = 4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24: |S_5| = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120:$$

- Look up the definition of adjacency in the text. Find the number of adjacencies in S_3 ; S_4 ; and S_5 :

group	total #adjacencies	average #adjacencies
S_3	8	$\frac{8}{6} = \frac{4}{3} = 1:333\ 3$
S_4	36	$\frac{36}{24} = \frac{3}{2} = 1:5$
S_5	192	$\frac{192}{120} = \frac{8}{5} = 1:6$

- What is the probability that a deck of 3, 4 or 5 cards will have an adjacency.

group	# of decks with an adjacency	prob of adjacency
S_3	6	$\frac{6}{6} = 1$
S_4	22	$\frac{22}{24} = \frac{11}{12} = :91667$
S_5	106	$\frac{106}{120} = \frac{53}{60} = :88333$