ECE-597: Probability, Random Processes, and Estimation
Homework # 5

Due: Thursday April 21, 2016

1) Consider the following Bernoulli Process

$$X[n] = \begin{cases} 1 & \text{for success in } n^{th} \text{ trial} \\ 0 & \text{for failure in } n^{th} \text{ trial} \end{cases}$$

with $P(X[n] = 1) = p$, and the $X[n]$ are i.i.d.
For this random sequence,
a) show that $\mu_X[n] = p$
b) show that $K_{XX}[k, l] = p(1 - p)\delta[k - l]$, where $\delta[k - l] = 1$ for $k = l$ and 0 for $k \neq l$.
c) Is this process WSS?

2) Consider the following Bernoulli Counting process

$$Y[n] = \sum_{i=1}^{n} X[i]$$

for the $X[i]$ defined in the previous problem.
a) Show that $\mu_Y[k] = kp$
b) Show that $K_{YY}[k, l] = p(1 - p)\min(k, l)$
c) Is this process WSS?

3) Consider the following random sequence.

$$Z[n] = \begin{cases} +1 & \text{for success in } n^{th} \text{ trial} \\ -1 & \text{for failure in } n^{th} \text{ trial} \end{cases}$$

where $P(Z[n] = 1) = p$, and the $Z[i]$ are i.i.d.
a) Show that $\mu_Z[n] = 2p - 1$
b) Show that $K_{ZZ}[k, l] = 4p(1 - p)\delta[k - l]$
c) Is this process WSS?

4) Consider the following random walk process

$$W[n] = \sum_{i=1}^{n} Z[i]$$

where the $Z[i]$ are defined in the previous problem.
a) Show that $\mu_W[n] = n(2p - 1)$
b) Show that $K_{WW}[k, l] = 4p(1 - p)\min(k, l)$
c) Is this process WSS?

5) Let $Z[n]$ be a one sided Bernoulli process with $p = 1/2$. This means that $Z[n]$ is an i.i.d. sequence with $P(Z[n] = 0) = P(Z[n] = 1) = 1/2$. Let

$$\begin{aligned} X[n] &= (-1)^{Z[n]} \\ Y[n] &= \sum_{i=0}^{i=n} 2^{-i} X[i] \\ V[n] &= \sum_{i=0}^{i=\infty} 2^{-i} X[n-i] \end{aligned}$$

Compute the mean and autocovariance functions of $X[n]$, $Y[n]$, and $Z[n]$. Are any of these WSS sequences?
Answer: $\mu_X[n] = 0$, $K_{XX}[n,m] = \delta[n-m]$, $\mu_Y[n] = 0$, $K_{YY}[n,m] = \frac{4}{3}\left(1 - (\frac{1}{4})^{\min(n,m)+1}\right)$, $\mu_V[n] = 0$, $K_{VV}[n,m] = \frac{4}{3}2^{-|n-m|}$