Name:

ECE 597: Probability, Random Processes, and Estimation

Exam #2

Friday May 6, 2016

Calculators may only be used for simple calculations.

Possible Useful Equations

$$
\begin{aligned}
y[n] &= \sum_{k=-\infty}^{\infty} h[k]w[n-k] \\
\sum_{m=-\infty}^{\infty} \gamma^m u[m]u[N-m] &= \frac{1-\gamma^{N+1}}{1-\gamma} \\
Z\left\{b^{|l|}\right\} &= \frac{1-b^2}{(1-bz^{-1})(1-bz)} \\
Z\left\{x[n-k]\right\} &= z^{-k}X(z) \\
S_{\mathbf{XX}}(z) &= K_o H_c(z) H_c(z^{-1}) \\
H_{Wiener}(z) &= \frac{1}{K_o}\frac{1}{H_c(z)}\left[\frac{S_{dx}(z)}{H_c(z^{-1})}\right]_+ \\
\epsilon^2 &= R_{dd}[0] - \sum_{m=0}^{\infty} h[m]R_{dx}[m]
\end{aligned}
$$

$$
\frac{f_{R|H_1}(r|H_1)}{f_{R|H_0}(r|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P_0(C_{10}-C_{00})}{P_1(C_{01}-C_{11})}
$$

You may assume all random variables and sequences are real valued.

1)Assume $X[n]$ is an i.i.d random sequence taking on values 1 and 0, with

$$P\{\mathbf{X}[n]=1\} = \frac{1}{3}$$
$$P\{\mathbf{X}[\mathbf{n}]=\mathbf{0}\} = \frac{2}{3}$$

Define then a new random sequence $\mathbf{Z}[n]$, where

$$\mathbf{Z}[n] = \sum_{k=1}^{k=n} \mathbf{X}[k]$$

Determine $\mu_{\mathbf{X}}[n]$, $\mu_{\mathbf{Z}}[n]$, $R_{\mathbf{XX}}[n,m]$, and $R_{\mathbf{ZZ}}[n,m]$

2) Consider a signal $S[n]$ with additive white noise $\mathbf{V}[n]$, so the input to the system is

$$\mathbf{X}[n] = S[n] + \mathbf{V}[n]$$

Assume $\mathbf{V}[n]$ is a zero mean process which is uncorrelated with $S[n]$. The correlations $R_{SS}[n]$ and $R_{\mathbf{VV}}[n]$ are given as

$$\begin{aligned} R_{SS}[n] &= 0.3^{|n|} \\ R_{\mathbf{VV}}[n] &= 0.5\delta[n] \end{aligned}$$

Determine the optimal linear (causal and stable) estimator for $S[n+1]$ using the Wiener filter. You can assume

$$S_{\mathbf{XX}}(z) = \frac{1.440(1 - 0.1042z)(1 - 0.1042z^{-1})}{(1 - 0.3z)(1 - 0.3z^{-1})}$$

3) Assume we have the zero mean WSS random sequence $\mathbf{X}[n]$. Assume we want to estimate $\mathbf{Y}[n]$ using the estimator

$$\hat{Y}[n] = a\mathbf{X}[n] + b\mathbf{X}[n-1]$$

Determine a matrix equation to be solved to determine a and b using the principle of orthogonality (the error is proportional to the data used to make the estimator). Your matrix equation will be in terms of $R_{\mathbf{XX}}$ and $R_{\mathbf{YX}}$.

4) Consider a signal $S[n]$ with additive white noise $\mathbf{V}[n]$, so the input to the system is

$$\mathbf{X}[n] \quad = \quad S[n] + \mathbf{V}[n]$$

where

- $S[n] = a^n u[n]$ is a **deterministic, not random** signal
- $\mathbf{V}[n]$ is zero mean white noise with $R_{\mathbf{VV}}[n,m] = \sigma^2 \delta[n-m]$
- the impulse response of the system is given by $h[n] = b^n u[n]$.

Determine the expected output power $E\left\{\mathbf{Y}^2[n]\right\}$

Hints:

- Assume you use the form $y[n] = \sum h[k]x[n-k]$
- You will need to be sure to use two sums with two dummy indices
- It is probably easier if you write $R_{\mathbf{XX}}[n,m]$ instead of using a single argument