ECE-597: Probability, Random Processes, and Estimation
Computer Project #5

Due: Friday April 22, 2016

In this project we will implement Autoregressive (AR), Moving Average (MA), and Autoregressive Moving Average (ARMA) random sequences. Since both the AR and MA are subsets of the ARMA sequences, we can just implement the ARMA sequence and then modify coefficients. We will also estimate the autocorrelation function $R_{xx}(r)$ assuming these are WSS sequences.

One general form of a second order ARMA discrete-time sequence is

$$x[n] = a_1 x[n-1] + a_2 x[n-2] + b_0 v[n] + b_1 v[n-1]$$

where $v[n]$ is a white noise process (i.i.d) with zero mean and known variance σ_v^2.

We can estimate the autocorrelation with *lag* r of the random sequence $x[n]$ as

$$R_{xx}(r) \quad = \quad \frac{1}{N-r} \sum_{n=1}^{n=N-r} x[n]x[n+r], \quad r = 0, 1, ..., M$$

One way to easily implement the estimate of the autocorrelation for lag r in Matlab is something like

```
r = 20;
R_xx = zeros(1,r);
for k = 0:r-1
  Rxx(k+1) = sum(x(1:N-k).*x(k+1:N))/(N-k);
end;
```

Remember that Matlab will not allow you to start with an index of 0, so you will need to include the shift when you plot the results (Rxx(1) actually is Rxx(0)).

When you plot the results, you should plot the random sequence as though it is a continuous function of the sample (don't try to plot as discrete points), and you should plot the autocorrelation estimate using the stem command (as discrete points) since we will not be looking at many lags.

For the following problems, assume we want $N = 1000$ sample points in the random sequence, and that all necessary initial conditions are set equal to zero (the easiest way to do this is just set $X = zeros(1, N)$).

1) Implement the AR sequence

$$x[n] = -0.9025x[n-2] + v[n]$$

assuming $\sigma_v^2 = 1$ and $v[n]$ is a Gaussian random variable. Plot the autocorrelation estimate for the first 20 lags. Run your code twice and turn in both plots. Note that each time we run the code we are getting a different *realization* of the noise sequence $v[n]$.

2) Implement the MA sequence

$$x[n] = v[n] - 0.5v[n-1]$$

assuming $\sigma_v^2 = 1$ and $v[n]$ is a Gaussian random variable. Plot the autocorrelation estimate for the first 20 lags. Run your code twice and turn in both plots. Note that each time we run the code we are getting a different *realization* of the noise sequence $v[n]$.

3) Implement the ARMA sequence

$$x[n] = 0.8x[n-1] + v[n] - 0.5v[n-1]$$

assuming $\sigma_v^2 = 1$ and $v[n]$ is a Gaussian random variable. Plot the autocorrelation estimate for the first 20 lags. Run your code twice and turn in both plots. Note that each time we run the code we are getting a different *realization* of the noise sequence $v[n]$.

Your write-up should be short and neat, and you should include a copy of your code. Your autocorrelation functions should look very much like those in the attached sheets.