ECE-597: Probability, Random Processes, and Estimation
Homework # 7

Due: Friday May 8, 2015
Exam 2: Friday May 8, 2015

Last week you derived the z- transform pair

$$a^{|m|} \quad \leftrightarrow \quad \frac{1-a^2}{(1-az^{-1})(1-az)}$$

which may be useful in the following problems.

1) Assume we have an autoregressive model

$$x[n] = ax[n-1] + v[n]$$

with $|a| < 1$, $v[n]$ zero mean white noise. Las weak you showed

$$S_{\mathbf{XX}}(z) \quad = \quad \frac{\sigma_v^2}{(1-az^{-1})(1-az)}$$

a) Assume our desired signal is $d[n] = x[n+1]$ (prediction). Show that the Wiener filter for this yields $\hat{d}[n] = ax[n]$.
b) Assume our desired signal is $d[n] = x[n+2]$. Show that the Wiener filter for this yields $\hat{d}[n] = a^2x[n]$.

2) Assume we have a moving average process

$$x[n] = v[n] - bv[n-1]$$

with $|b| < 1$, and $v[n]$ zero mean white noise. Last weak you showed that

$$S_{\mathbf{XX}}(z) = \sigma_v^2(1-bz^{-1})(1-bz)$$

Assume we want to estimate $d[n] = x[n+1]$. Show that the Wiener filter for this is given by $h_o[n] = -b^{n+1}u[n]$.

3)Assume we have a simple autoregressive moving average model

$$x[n] = ax[n-1] - bv[n-1] + v[n]$$

with $|a| < 1, |b| < 1$. Last week you showed that

$$S_{\mathbf{XX}}(z) = \sigma_v^2 \frac{(1 - bz^{-1})(1 - bz)}{(1 - az^{-1})(1 - az)}$$

a) Assume we want the estimate $d[n] = x[n+1]$. Show that the Weiner filter for this is given by $h_o[n] = (a - b)b^n u[n]$.
b) Assume we want the estimate $d[n] = x[n+2]$. Show that the Wiener filter for this is given by $h_o[n] = a(a - b)b^n u[n]$.

4) Consider as signal $S[n]$ plus noise $V[n]$ process with observed signal $X[n] = S[n] + V[n]$ where $V[n]$ is a zero mean white noise signal and

$$\begin{aligned} S_{\mathbf{SS}}(z) &= \frac{0.36}{1.04 - 0.2z - 0.2z^{-1}} \\ S_{\mathbf{VV}}(z) &= 1 \end{aligned}$$

If the desired signal is $d[n] = S[n]$, show that the Wiener filter is given by $h_o[n] = 0.271(0.146)^n u[n]$.

5) Consider as signal $S[n]$ plus noise $V[n]$ process with observed signal $X[n] = S[n] + V[n]$ where $V[n]$ is a zero mean white noise signal and

$$\begin{aligned} R_{\mathbf{SS}}[m] &= 1.2(0.7)^{|m|} \\ R_{\mathbf{VV}}[m] &= 0.2\delta[m] \end{aligned}$$

a) If the desired signal is $d[n] = S[n]$, show that the Wiener filter is given by $h_o[n] = 0.775(0.158)^n u[n]$ and the mean squared error is 0.154.
b) If the desired signal is $d[n] = S[n+1]$ show that the Wiener filter is given by $h_o[n] = 0.542(0.158)^n u[n]$ and the mean squared error is 0.688.