ECE-597: Probability, Random Processes, and Estimation
Homework # 6

Due: Friday May 1, 2015

From the textbook: 8.24, 8.26 (a,b), 8.34, 8.35, 8.37, 8.42

Hints and Answers:
8.24, $K_{\mathbf{YY}}[n] = \sigma^2 \left\{ (1+|\alpha|^2)\rho^{|n|} - \alpha^* \rho^{|n+1|} - \alpha\rho^{|n-1|} \right\}$, $\alpha = \rho$ or $\alpha = 1/\rho$
8.26, $\mu_{\mathbf{y}}[n] = \frac{5}{1-\rho}$, $E[y[n]^2] = \frac{25}{(1-\rho)^2} + \frac{\sigma_w^2}{1-\rho^2}$
8.34, $h[m] = K_{\mathbf{XW}}[m]$, $S_{\mathbf{XW}}(\omega) = H(\omega)$
8.35, $S_{\mathbf{XY}}(\omega) = H^*(\omega) S_{\mathbf{XX}}(\omega)$
8.37, $S_{\mathbf{XW}}(\omega) = \frac{S_{\mathbf{XX}}(\omega)}{\sqrt{S_{\mathbf{YY}}(\omega)}}$, $h[n] = R_{\mathbf{XW}}[n]$ for $n = 0, 1, ..., N-1$
8.42, $\mu_{\mathbf{X}}[n] = \frac{n}{2}(s_1 - s_2)$, $R_{\mathbf{XX}}[n_1, n_2] = \frac{min(n_1,n_2)}{4}(s_1+s_2)^2 + \frac{n_1 n_2}{4}(s_1 - s_2)^2$

Additional Problem

a) We can write $a^{|m|}$ as

$$a^{|m|} \;=\; a^m u[m-1] + a^{-m} u[-m]$$

Using this relationship derive the z- transform pair

$$a^{|m|} \;\leftrightarrow\; \frac{1-a^2}{(1-az^{-1})(1-az)}$$

b) Assume we have an autoregressive model

$$x[n] = ax[n-1] + v[n]$$

with $|a| < 1$, $v[n]$ zero mean white noise. Show that

$$S_{\mathbf{XX}}(z) \;=\; \frac{\sigma_v^2}{(1-az^{-1})(1-az)}$$

$$R_{\mathbf{XX}}[m] \;=\; \frac{\sigma_v^2 a^{|m|}}{1-a^2}$$

c) Assume we have a moving average process

$$x[n] = v[n] - bv[n-1]$$

with $|b| < 1$, and $v[n]$ zero mean white noise. Show that

$$S_{\mathbf{XX}}(z) = \sigma_v^2 (1 - bz^{-1})(1 - bz)$$

$$R_{\mathbf{XX}}[m] = (1+b^2)\sigma_v^2 \delta[m] - b\sigma_v^2 \delta[m+1] - b\sigma_v^2 \delta[m-1]$$

d) Assume we have a simple autoregressive moving average model

$$x[n] = ax[n-1] - bv[n-1] + v[n]$$

with $|a| < 1, |b| < 1$.Show that

$$S_{\mathbf{XX}}(z) = \sigma_v^2 \frac{(1-bz^{-1})(1-bz)}{(1-az^{-1})(1-az)}$$

and find $R_{\mathbf{XX}}[m]$