ECE-597: Probability, Random Processes, and Estimation
Computer Project #4

Due: April 17, 2015

In this project we will build on the results from Project 3, but now we will accept hypotheses using the likelihood ratio test and incorporate *a priori* information.
From Homework 4, if we have Gaussian distributed two random vectors $\mathbf{Y}_1$ and $\mathbf{Y}_2$ with means $\underline{\mu}_2$ and $\underline{\mu}_1$ and covariance matrices K_2 andK_1, the likelihood ratio test is

$$\frac{1}{2}(\underline{r}-\underline{\mu}_2)^T K_2^{-1}(\underline{r}-\underline{\mu}_2) - \frac{1}{2}(\underline{r}-\underline{\mu}_1)^T K_1^{-1}(\underline{r}-\underline{\mu}_1) \underset{H_2}{\overset{H_1}{\gtrless}} \ln(\frac{P_2}{P_1}) + \frac{1}{2}\ln|K_1| - \frac{1}{2}\ln|K_2|$$

The first four parts below are what you did for project 3, they are just repeated here.
1) Determine the matrix A such that if $\underline{\mathbf{X}}$ is a random vector with zero mean, and uncorrelated components with

$$K_{\mathbf{X}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then the random variable $\underline{\mathbf{Y}}_1 = A\underline{\mathbf{X}} + \underline{\mu}_{\mathbf{Y}_1}$ will have covariance matrix

$$K_{\mathbf{Y}_1} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 5 \end{bmatrix}$$

and mean $\underline{\mu}_{\mathbf{Y}_1}$

2) Using Matlab, generate 1000 2-dimensional random vectors ($\underline{X}$'s) with uncorrelated components, each component with an $N(0,1)$ distribution using the command

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X=randn(3,1000)
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and construct the random variables $\underline{\mathbf{Y}}_1$ so they will have covariance $K_{\mathbf{Y1}}$ and mean

$$\underline{\mu}_{\mathbf{Y}_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

3) Now construct random variables $\underline{\mathbf{Y}}_2$ with 1000 samples and with covariance and mean

$$K_{\mathbf{Y}_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 1 \\ 1 & 1 & 5 \end{bmatrix}, \underline{\mu}_{\mathbf{Y}_2} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

4) To determine if your transformation is correct, estimate the means and covariances using the (unbiased) estimators

$$\begin{aligned}
\underline{\hat{\mu}} &= \frac{1}{N}\sum_{i=1}^{i=N} \underline{\mathbf{Y}}(i) \\
\hat{K} &= \frac{1}{N-1}\sum_{i=1}^{i=N} (\underline{\mathbf{Y}}(i) - \underline{\hat{\mu}})(\underline{\mathbf{Y}}(i) - \underline{\hat{\mu}})^T
\end{aligned}$$

You may want to use a larger sample size at this point to be sure your estimators are converging to the correct values.

5) Now we want to use the likelihood ratio test to do the hypothesis testing. Specifically I want you to do the following:

- Generate 700 realizations of $\mathbf{Y}_1$ (with *a priori* probability $P_1 = 0.7$) and 300 realizations of $\mathbf{Y}_2$ (with *a priori* probability $P_2 = 0.3$).
- Utilize the likelihood ratio test and determine how many correct decisions were made (how many times did the program choose H_1 when it was correct, how many times did your program choose H_1 when it was wrong, how many times did it choose H_2 when it was correct, and how many times did it choose H_2 when it was wrong.
- Apply the likelihood ratio test again, but assume both *a priori* probabilities are (incorrectly) set to 0.5. Be sure to use the same data as in the previous part, **do not** generate new data. Again, record all the correct and incorrect decisions made.

Your write-up should be short and neat, and you should include a copy of your code.