ECE-597: Optimal Control Homework #8

1) From the text, 5.2.1.

2) From the text, 5.2.3. Don't follow the solution. Use the equation

$$\dot{P} \;=\; AP + PA^T - BR^{-1}B^T$$

and then the equations

$$\begin{aligned} \dot{M} &= -MA \\ \frac{d}{dt}[\mathcal{Q}(t)]^{-1} &= = -MBR^{-1}B^TM^T \end{aligned}$$

to find $K(t)$ in terms of $T = t_f - t$. This is "all" you have to do for this problem.

3) Consider the tracking problem

$$\begin{aligned} \text{minimize } J &= \frac{1}{2}(y(T) - r(T))^T P(y(T) - r(T)) + \frac{1}{2}\int_{t_0}^{T} \left[(y - r)^T Q(y - r) + u^T Ru\right] dt \\ \text{subject to} \quad & \dot{x} = Ax + Bu \\ & y(t) = Cx(t) \\ & P \geq 0,\ Q \geq 0,\ R > 0,\ P = P^T,\ Q = Q^T,\ R = R^T \end{aligned}$$

Here $r(t)$ is a reference signal we would like to track. Show that the solution to this problem can be written as:

$$\begin{aligned} \dot{S} &= -SA - A^TS + SBR^{-1}B^TS - C^TQC, \quad S(T) = C^TPC \\ \dot{g} &= -(A - BK)^Tg + C^TQr, \quad g(T) = -C^TPr(T) \\ K(t) &= R^{-1}B^TS \\ u(t) &= -Kx - R^{-1}B^Tg \end{aligned}$$