ECE-597: Optimal Control Homework #2

1) Consider the problem:

$$\begin{array}{rc} \text{minimize} & x_1 x_2 - 2x_1 \\ \text{subject to} & x_1^2 - x_2^2 = 0 \end{array}$$

a) Show that if a solution exists ($H_x = 0$), it must be either $[1, 1]^T$ or $[-1, 1]^T$
b) Use the second oder sufficient conditions

$$\Delta x^T H_{xx} \Delta x > 0$$

for all Δx such that $f_x \Delta x = 0$ to show that $[1, 1]^T$ is the minimizer (and $[-1, 1]^T$ does not meet the sufficiency conditions).

2) Consider the problem of minimizing

$$\Pi = \frac{x^T Q x}{x^T P x}$$

where $Q = Q^T > 0$ and $P = P^T > 0$.
a) Show that if z is a solution, then so is tz for all nonzero scalars t. Hence to avoid multiplicity of solutions, we impose the constraint

$$x^T P x = 1$$

b) Now solve the problem

$$\begin{array}{rc} \text{minimize} & x^T Q x \\ \text{subject to} & x^T P x - 1 = 0 \end{array}$$

In particular, show that the vectors x that solve this problem are eigenvectors that solve the following eigenproblem:

$$P^{-1} Q x = \lambda x$$

and then show that the maximum value of $x^T Q x$ is in fact equal to the largest eigenvalue of this eigenproblem.

3) Consider the sequence $\{x_k\}$ generated by

$$x_{k+1} = ax_k + bu_k$$

where a and b are real and nonzero. In particular, we want to find u_0 and u_1 such that x_2 is zero and the average input energy $\frac{1}{2}(u_0^2 + u_1^2)$ is minimized.
a) Find expressions for u_0 and u_1 in terms of a, b, x_0.
b) Look at the second order conditions to determine if you do, indeed, have a minimum.

Problem 2.1.3
a) You do not need to compute g_θ or g_b
b) The solutions are somewhat misleading in their initial description of $H(i)$. Be careful to explain what you are doing!
c) Don't run the code, just do the analytical part.

Problem 2.1.4
a) Be sure to explain how you got the dimensionless equations! Probably best to do this at the end of part (a).
b) The second equation in the book ($H_{\theta(i)}$) is correct, I think.
c) Don't run the code, just do the analytical part.