ECE-521 Control Systems II

Homework 4

Due at the beginning of class, Tuesday January 11 , 2005
No Lab Tuesday January 11, 2005
Exam 1, Thursday January 13, 2005

1) Consider the following state variable model:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = [1 \quad 0] x(t)$$

a) Is the system controllable? Is the system observable?

b) Assume state variable feedback of the form $u(t) = G_{pf} r(t) - kx(t)$. Show that the transfer function for the system with state variable feedback is given by

$$G(s) = C(sI - \tilde{A})^{-1}\tilde{B} = \frac{G_{pf}}{s^2 + (k_2 - 1)s + (k_1 - 1)}$$

c) Using the results from part (b), determine the values of k_1 and k_2 so the closed loop poles are at $-1 \pm j$.

d) Use the transformation matrix T method (page 833 of text) to compute the required values of k_1 and k_2 so the closed loop poles are at $-1 \pm j$.

e) Use Ackerman's formula to compute the required values of k_1 and k_2 so the closed loop poles are at $-1 \pm j$.

2) Consider the following state variable model:

$$\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = [0 \quad 1] x(t)$$

a) Is the system controllable? Is the system observable?

b) Assume state variable feedback of the form $x(t) = G_{pf} r(t) - kx(t)$. Show that the transfer function for the system with state variable feedback is given by

$$G(s) = C(sI - \tilde{A})^{-1}\tilde{B} = \frac{G_{pf}(s-2)}{s^2 + (k_2 - 2)s + (k_1 - 2k_2 - 1)}$$

c) Using the results from part (b), determine the values of k_1 and k_2 so the closed loop poles are at $-1 \pm j$.

d) Use transformation matrix T method (page 833 of text) to compute the required values of k_1 and k_2 so the closed loop poles are at $-1 \pm j$.

e) Use Ackerman's formula to compute the required values of k_1 and k_2 so the closed loop poles are at $-1 \pm j$.

3) Consider a the following state variable model

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$
$$y(t) = [1 \quad 0] x(t)$$

How many linearly independent columns are there in the controllability matrix? In the observability matrix? Is the system controllable? Is the system observable?

4) Consider a the following state variable model

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x(t)$$

How many linearly independent columns are there in the controllability matrix? In the observability matrix? Is the system controllable? Is the system observable?