ECE-521 Control Systems II
Homework 7

Due Date: Thursday May 20

1) For a system with plant

$$G_p(s) = \frac{s+3}{s(s-1)}$$

show that the quadratic optimal closed loop transfer function is

$$G_0(s) = \frac{10(s+3)}{s^2+12.7s+30}$$

when $q = 100$.

2) For a system with plant

$$G_p(s) = \frac{s-1}{s(s-2)}$$

show that the quadratic optimal closed loop transfer function is

$$G_0(s) = \frac{-10(s-1)}{s^2+11.1s+10}$$

when $q = 100$.

3) For a one degree of freedom system like we have in lab, with plant

$$G_p(s) = \frac{15}{0.0025s^2+0.0080s+1}$$

a) Show that when $q = 0.1$ the quadratic optimal closed loop transfer function is

$$G_0(s) = \frac{1856.6}{s^2+55.5s+1939.1}$$

b) Show that the controller is given by

$$G_c(s) = \frac{0.0038s^2+0.012s+1.5}{0.012s^2+0.67s+1}$$

For the following three problems, it is useful to remember

- our compensator has the form $G_c(s) = \frac{B(s)}{A(s)}$, where $B(s) = B_0 + B_1 s + ...$ and $A(s) = A_0 + A_1 s + ...$
- the plant has the form $G_p(s) = N(s)/D(s)$, where $N(s) = N_0 + N_1 s + ...$ and $D(s) = D_0 + D_1 s + ...$
- The desired characteristic polynomial for the closed loop transfer function is $D_0(s) = F_0 + F_1 s + F_2 s^2 + ...$
- To determine the equations to solve set

$$A(s)D(s) + B(s)N(s) \quad = \quad D_0(s)$$

  and equate coefficients of s. This will give you the system of equations to solve.

- To use the compensator to make a system a type one system, set $A_0 = 0$

4) For the plant

$$G_p(s) \quad = \quad \frac{1}{s(s+2)}$$

show that the first order compensator that will put the closed loop poles at $-1 \pm j$ and -3 is $G_c(s) = 2$.

5) For the plant

$$G_p(s) \quad = \quad \frac{1}{s+2}$$

Assume we want to place both closed loop poles at -4 and also have a type 1 system. Show that the first order compensator that will do this is given by

$$G_c(s) \quad = \quad \frac{6(s+2.667)}{s}$$

6) For a plant like the systems we have in lab with transfer function given by

$$G_p(s) \quad = \quad \frac{6000}{s^2 + 3.2s + 400}$$

a) Show that the first order compensator that places all three closed loop poles at -5 is given by

$$G_c(s) \quad = \quad \frac{-0.0605s - 0.7658}{s + 11.8}$$

Note that the numerator of this compensator make it pretty useless for tracking a step input.

b) Show that the second order system that places all four closed loop poles at -5 and produces a type 1 system is given by

$$G_c(s) \quad = \quad \frac{-0.5063s^2 - 1.0367s + 0.1042}{s(s + 16.8)}$$