ECE-520 Control Systems II
Homework 3

<u>Due Date:</u> Tuesday March 30

If matrix P is given as

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Then

$$P^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

and the *determinant* of P is given by $ad - bc$.

The general for for writing a continuous time state variable system is

$$\begin{aligned} \dot{\underline{x}}_(t) &= A\underline{x}(t) + B\underline{u}(t) \\ \underline{y}(t) &= C\underline{x}(t) + D\underline{u}(t) \end{aligned}$$

Now assume we are using state variable feedback, so that $\underline{u}(t) = f\underline{r}(t) - \underline{k}^T\underline{x}(t)$. Here $\underline{r}(t)$ is our new reference input, f is a scaling factor, and $k^T = [k_1 \; k_2]$ is the feedback gain matrix. With this state variable feedback, we have the system

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B(f\underline{r}(t) - \underline{k}^T\underline{x}(t))$$

or

$$\dot{\underline{x}}(t) = \tilde{A}\underline{x}(t) + \tilde{B}\underline{r}(t)$$

where $\underline{r}(t)$ is the new input. For $D = 0$, the transfer matrix is given by

$$G(s) = C\left[(sI - \tilde{A})^{-1}\right]\tilde{B}$$

For each of the systems below,

- determine the controllability matrix and if the system is controllable
- determine the observability matrix and if the system is observable
- determine the transfer matrix
- determine the transfer matrix when there is state variable feedback
- determine if k_1 and k_2 exist to allow us to place the poles arbitrarily

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Let

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [0\ 1], D = 0$$

Not Controllable, Observable, $G(s) = \frac{(s-1)f}{(s-1)(s-1+k_2)}$

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Let

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [0\ 1], D = 0$$

Controllable, Not Observable, $G(s) = \frac{sf}{s^2+(k_2-1)s+k_1}$

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Let

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1\ 0], D = 0$$

Controllable, Observable, $G(s) = \frac{f}{s^2+(k_2-1)s+(k_1-1)}$

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Let

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = [0\ 1], D = 0$$

Not controllable, Observable, $G(s) = \frac{(s+1)f}{(s+k_1)(s+k_2)-(k_1-1)(k_2-1)}$