

ECE-520 Lab 2

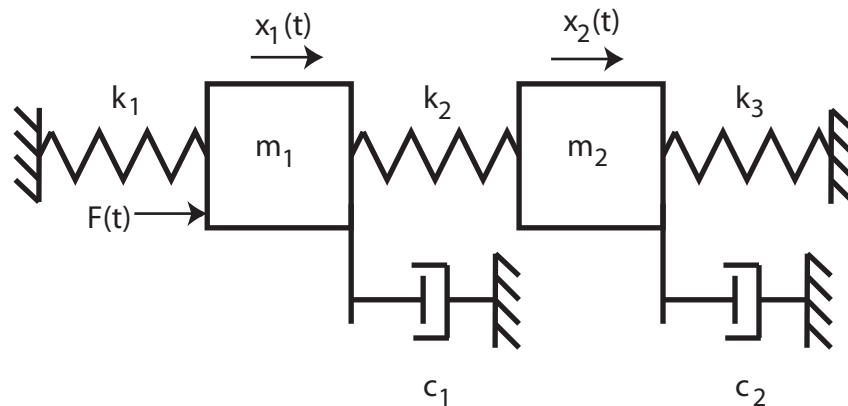
Time and Frequency Domain Modeling of Two Degree of Freedom Systems

Overview

In this lab, you will be modeling two two degree of freedom system using time-domain analysis and frequency domain analysis. You will model one rectilinear and one torsional system. The steps we will go through in this lab are very commonly used in system identification (determining the transfer function) of unknown systems. We will utilize these models in later labs so do a good job in this lab, your results in later labs will be affected by how well you perform in this lab. **Take your time!**

Background

For the following rectilinear two degree of freedom configuration (one of the springs many be missing

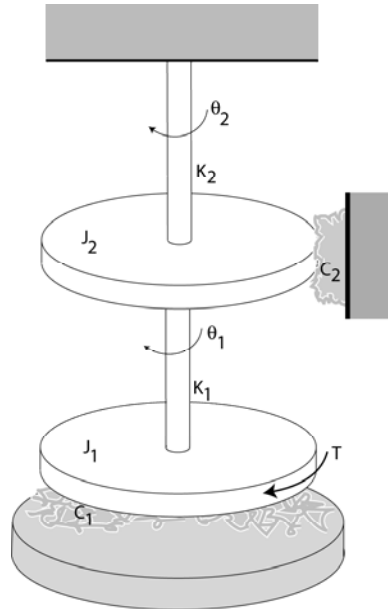


we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{k_1+k_2}{m_1}\right) & -\left(\frac{c_1}{m_1}\right) & \left(\frac{k_2}{m_1}\right) & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{k_2}{m_2}\right) & 0 & -\left(\frac{k_2+k_3}{m_2}\right) & -\left(\frac{c_2}{m_2}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{1}{m_1}\right) \\ 0 \\ 0 \end{bmatrix} F$$

We need to identify all of these quantities to get the A and B matrices for the state variable description. For our system $D=0$ and C is determined by whatever we want the output to be.

For the following torsional two degree of freedom configuration



we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{k_1}{J_1}\right) & -\left(\frac{c_1}{J_1}\right) & \left(\frac{k_1}{J_1}\right) & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{k_1}{J_2}\right) & 0 & -\left(\frac{k_1+k_2}{J_2}\right) & -\left(\frac{c_2}{J_2}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{1}{J_1}\right) \\ 0 \\ 0 \end{bmatrix} T$$

We need to identify all of these quantities to get the A and B matrices for the state variable description. For our system $D=0$ and C is determined by whatever we want the output to be. For both of these systems, we can determine the parameters we need by fitting the frequency response data to the transfer functions

$$\frac{X_2(s)}{F(s)} = \frac{K_2}{\left(\frac{1}{\omega_a^2} s^2 + \frac{2\zeta_a}{\omega_a} s + 1\right) \left(\frac{1}{\omega_b^2} s^2 + \frac{2\zeta_b}{\omega_b} s + 1\right)}$$

$$\frac{X_1(s)}{F(s)} = \frac{\frac{1}{m_1} (s^2 + 2\zeta_2 \omega_2 s + \omega_2^2)}{(s^2 + 2\zeta_a \omega_a s + \omega_a^2) (s^2 + 2\zeta_b \omega_b s + \omega_b^2)}$$

You will need to set up a folder for Lab 2 and copy all files from the class website into this folder, as well as the files you used for Lab 1.

Part A: Modeling the Rectilinear System

You need to fill out the data sheet indicating the configuration on the last page of the lab and turn it in!

Step 0: Set Up the System. Both the first and second carts should move. The third cart should be fixed in place. In addition:

- Either the carts should have an equal amount of weight on them, or the first cart should have more weight than the second cart. You need at least one mass on each cart.
- Either all springs connecting carts should have equal stiffness, or the springs should get less stiff from left to right. You need to use at least two springs.
- If you want to use the active damper, unscrew the screw in the damper.

You will be using this configuration throughout the remainder of the course so be sure you write down all of the information you need to duplicate this configuration.

Step 1a) Initial Estimates of ω_1 , ζ_1 , ω_2 , and ζ_2

From the equations of motion we have

$$\begin{aligned}\ddot{x}_1 + 2\zeta_1\omega_1\dot{x}_1 + \omega_1^2x_1 &= \frac{k_2}{m_1}x_2 + \frac{1}{m_1}F \\ \ddot{x}_2 + 2\zeta_2\omega_2\dot{x}_2 + \omega_2^2x_2 &= \frac{k_2}{m_2}x_1\end{aligned}$$

a) If there is no applied force ($F = 0$) and the second cart is fixed in place ($x_2 = 0$), we have

$$s^2 + 2\zeta_1\omega_1s + \omega_1^2 = 0$$

Use the log decrement method to get our initial estimate of ω_1 and ζ_1 .

b) If the second cart is free to move and the first cart is fixed in place ($x_1 = 0$), we have

$$s^2 + 2\zeta_2\omega_2s + \omega_2^2 = 0$$

Use the log decrement method to get our initial estimate of ω_2 and ζ_2 .

For the log-decrement analysis you will go through the following steps for **each** cart:

- Be sure only one cart is free to move
- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model210_Openloop.mdl** so the input has **zero** amplitude.
- Compile **Model210_Openloop.mdl** if necessary.
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Displace whichever cart is free to move and hold it.
- Start (**play**) **Model210_Openloop.mdl** and let the cart go.
- Run the m-file **log_dec.m**. This should be in the same directory as **Model210_Openloop.mdl** and **log_dec.fig**. This routine assumes the position of the first cart is labeled $x1$, the position of the second cart is labeled $x2$, and the time is labeled $time$. (These are the defaults in **Model210_Openloop.mdl**.)

You will need to include these **two** log-decrement results in your memo.

Step 1b) Estimating the Gains K_1 and K_2

You will go through the following steps:

- Be sure both carts are free to move
- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model210_Openloop.mdl** so the input is a step. Set the amplitude to something small, like 0.01 or 0.02 cm.
- Compile **Model210_Openloop.mdl**
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl**. If the carts do not seem to move much, increase the amplitude of the step. If the carts move too much, decrease the amplitude of the step. You may have to recompile.
- Estimate the static gains as

$$K_1 = \frac{x_{1,ss}}{A} \quad K_2 = \frac{x_{2,ss}}{A}$$

where x_{ss} is the steady state value of the cart position, and A is the input amplitude.

You should do this in Matlab, don't use the X-Y Graph. The variables $x1$, $x2$, and $time$ should be in your workspace.

You need to increase the value of the input amplitude until the first cart is moving about 2 cm or so Use the static gains associated with this input amplitude as your estimates of the static gain.

Step 1c) Fitting the Estimated Frequency Response to the Measured Frequency Response

We will be constructing the magnitude portion of the Bode plot and fitting this measured frequency response to the frequency response of the expected transfer function to determine the parameters we need. For each frequency $\omega = 2\pi f$ we have as input $u(t) = A \cos(\omega t)$ where, for our systems, A is measured in centimeters. After a transition period, the steady state output will be $x_1(t) = B_1 \cos(\omega t + \theta_1)$ for the first cart and $x_2(t) = B_2 \cos(\omega t + \theta_2)$ for the second cart, where both B_1 and B_2 are also measured in cm. Since we will be looking only at the magnitude portion of the Bode plot, we will ignore the phase angles θ_1 and θ_2 .

You will go through the following steps

For frequencies $f = 0.5, 1, 1.5 \dots 7.5$ Hz

- Make sure both carts are free to move, and the third cart is fixed.
- Modify **Model210_Openloop.mdl** so the input is a sinusoid. You may have to set the mode to **Normal**.
- Set the frequency and amplitude of the sinusoid. Try a small amplitude to start, like 0.01 cm. Generally this amplitude should be as large as you can make it without the system hitting a limit. This amplitude will probably vary with each frequency.
- Compile **Model210_Openloop.mdl**, if necessary. (Assume it is not necessary. The system will let you know if it is necessary.)
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl**. If the carts do not seem to move much, increase the amplitude of the input sinusoid. If the carts move too much, decrease the amplitude of the input sinusoid.
- Record the input frequency (f), the amplitude of the input (A), and the amplitude of the output (B_1 and B_2) when the system is in steady state. Modify the program **get_Amp2.m** to help record these amplitudes accurately. Be sure the plot from **get_Amp2** shows the system in steady state. Be sure to look at the graph and understand what the code is doing before you use it!!!
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Enter the values of f , A , B_1 and B_2 into the program **process_data_2dof.m** (you need to edit the file)

At the Matlab prompt, type **data = process_data_2dof;**

Run the program **model_2dof.m**. There are ten input arguments to this program:

- **data**, the measured data as determined by **process_data_2dof.m**
- the estimated value of K_2
- ω_a , the estimated frequency of the first resonance, when both carts are moving, in radians/sec
- ζ_a , the estimated first damping ratio when both carts are moving. Assume $\zeta_a = 0.1$.
- ω_b , the estimated frequency of the second resonance, when both carts are moving, in radians/sec
- ζ_b , the estimated second damping ratio when both carts are moving. Assume $\zeta_b = 0.1$.
- ω_1 the estimated natural frequency of the first cart when it is the only cart moving (from the log decrement analysis)
- ζ_1 the estimated damping ratio of the first cart when it is the only cart moving (from the log decrement analysis)
- ω_2 the estimated natural frequency of the second cart when it is the only cart moving (from the log decrement analysis)
- ζ_2 the estimated damping ratio of the second cart when it is the only cart moving (from the log decrement analysis)

The program **model_2dof.m** will produce the following:

- A graph indicating the fit of the identified transfer function to the measured data for the first cart (You need to include the *final* graph of this fit in your memo.)
- A graph indicating the fit of the identified transfer function to the measured data for the second cart (You need to include the *final* graph of this fit in your memo.)
- The optimal estimates of all parameters (written at the top of the graphs)
- A file **state_model_2dof.mat** in your directory. This file contains the A, B, C, and D matrices for the state variable model of the system. If you subsequently type **load state_model_2dof** you will load these matrices into your workspace.

You need to be sure you have 4 points close to the resonant peaks of the transfer functions. This is particularly true if you have very small values of ζ (which correspond to very sharp peaks) At this point you probably should go back and add a few points near both the resonant peaks and nulls.

Part B: Modeling the Torsional System

You need to fill out the data sheet indicating the configuration on the last page of the lab and turn it in!

Step 0: Set Up the System. Both the first and second disks should move. The third disk should be fixed in place. In addition:

- Either the disks should have an equal moment of inertia, or the first disk should have a larger moment of inertia than the second disk.
- The masses should be placed symmetrically on each disk, and the moment of inertia is generally given by mr^2

You will be using this configuration throughout the remainder of the course so be sure you write down all of the information you need to duplicate this configuration.

Step 1a) Initial Estimates of ω_1 , ζ_1

From the equations of motion for the first disk we have

$$\ddot{\theta}_1 + 2\zeta_1\omega_1\dot{\theta}_1 + \omega_1^2\theta_1 = \frac{k_1}{J_1}\theta_2 + \frac{1}{J_1}T$$

If there is no applied force torque ($T = 0$) and the second disk is fixed in place ($\theta_2 = 0$), we have

$$s^2 + 2\zeta_1\omega_1s + \omega_1^2 = 0$$

Use the log decrement method to get our initial estimate of ω_1 and ζ_1 .

Since we cannot lock the first disk in place, we cannot estimate these parameters for the second disk. We will initially assume they are the same for both disks.

For the log-decrement analysis you will go through the following steps:

- Be sure only one disk is free to move
- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model205_Openloop.mdl** so the input has zero amplitude.
- Compile **Model205_Openloop.mdl** if necessary.
- Connect **Model205_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Displace the first disk, and hold it. Don't displace the disk more than about 20 degrees.
- Start (**play**) **Model205_Openloop.mdl** and let the disk go.
- Run the m-file **log_dec.m**. This should be in the same directory as **Model205_Openloop.mdl** and **log_dec.fig**. This routine assumes the position of

the first disk is labeled *theta1* and the time is labeled *time*. (These are the defaults in **Model205_Openloop.mdl**.)

You will need to include this log-decrement result in your memo.

Step 1b) Estimating the Gains K_1 and K_2

You will go through the following steps:

- Be sure both disks are free to move
- Reset the system using **ECPDSPresetmdl.mdl**.
- Modify **Model205_Openloop.mdl** so the input is a step. Set the amplitude to something small, like 1 or 2 degrees (the ECP system expects the input in *radians*).
- Compile **Model205_Openloop.mdl**
- Connect **Model205_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model205_Openloop.mdl**. If the disks do not seem to move much, increase the amplitude of the step. If the disks move too much (more than 25 degrees), decrease the amplitude of the step. You may have to recompile.
- Estimate the static gains as

$$K_1 = \frac{\theta_{1,ss}}{A} \quad K_2 = \frac{\theta_{2,ss}}{A}$$

where θ_{ss} is the steady state value of the disk position, and A is the input amplitude.

Be sure your units are consistent! Either both in radians or both in degrees!

You should do this in Matlab, don't use the X-Y Graph. The variables *theta1*, *theta2*, and *time* should be in your workspace.

You need to increase the value of the input amplitude until the first disk is moving about 20 degrees or so (this is not the steady state value, it needs to move though.) Use the static gains associated with this input amplitude as your estimates of the static gain.

Step 1c) Fitting the Estimated Frequency Response to the Measured Frequency Response

We will be constructing the magnitude portion of the Bode plot and fitting this measured frequency response to the frequency response of the expected transfer function to determine the parameters we need. For each frequency $\omega = 2\pi f$ we have as input $u(t) = A \cos(\omega t)$ where, for our systems, A is measured in radians (or degrees). After a transition period, the steady state output will be $x_1(t) = B_1 \cos(\omega t + \theta_1)$ for the first disk and $x_2(t) = B_2 \cos(\omega t + \theta_2)$ for the second disk, where both B_1 and B_2 are also measured

in cm. Since we will be looking only at the magnitude portion of the Bode plot, we will ignore the phase angles θ_1 and θ_2 .

Limit the motion of the torsional system to +/- 30 degrees. If it appears to be turning more than this stop the system and put in a smaller amplitude!

You will go through the following steps

For frequencies $f = 0.5, 1, 1.5 \dots 7.5$ Hz

- Make sure both disks are free to move, and the third disk is fixed.
- Modify **Model205_Openloop.mdl** so the input is a sinusoid. You may have to set the mode to **Normal**.
- Set the frequency and amplitude of the sinusoid. Try a small amplitude to start, like 1 degree (the ECP expects the input in *radians*). Generally this amplitude should be as large as you can make it without the system hitting a limit or the system moving more than 25 degrees. This amplitude will probably vary with each frequency.
- Compile **Model205_Openloop.mdl**, if necessary. (Assume it is not necessary. The system will let you know if it is necessary.)
- Connect **Model205_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model205_Openloop.mdl**. If the disks do not seem to move much, increase the amplitude of the input sinusoid. If the disks move too much, stop the system and decrease the amplitude of the input sinusoid.
- Record the input frequency (f), the amplitude of the input (A), and the amplitude of the output (B_1 and B_2) **when the system is in steady state**. Modify the program **get_Amp2.m** to help record these amplitudes accurately. Be sure the plot from **get_Amp2** shows the system in steady state. Be sure to look at the graph and understand what the code is doing before you use it!!!

Enter the values of f , A , B_1 and B_2 into the program **process_data_2dof.m** (you need to edit the file). Note that you need to be consistent in your units for the amplitudes, either all in degrees (easy) or all in radians (hard).

At the Matlab prompt, type **data = process_data_2dof;**

Run the program **model_2dof.m**. There are ten input arguments to this program:

- **data**, the measured data as determined by **process_data_2dof.m**
- the estimated value of K_2
- ω_a , the estimated frequency of the first resonance, when both disks are moving, in radians/sec

- ζ_a , the estimated first damping ratio when both disks are moving. Assume $\zeta_a = 0.1$.
- ω_b , the estimated frequency of the second resonance, when both disks are moving, in radians/sec
- ζ_b , the estimated second damping ratio when both disks are moving. Assume $\zeta_b = 0.1$.
- ω_1 the estimated natural frequency of the first disk when it is the only disk moving (from the log decrement analysis)
- ζ_1 the estimated damping ratio of the first disk when it is the only disk moving (from the log decrement analysis)
- ω_2 the estimated natural frequency of the second disk when it is the only disk moving (assume its the same as for the first disk)
- ζ_2 the estimated damping ratio of the second disk when it is the only disk moving (assume its the same as for the first disk)

The program **model_2dof.m** will produce the following:

- A graph indicating the fit of the identified transfer function to the measured data for the first disk (You need to include the *final* graph of this fit in your memo.)
- A graph indicating the fit of the identified transfer function to the measured data for the second disk (You need to include the *final* graph of this fit in your memo.)
- The optimal estimates of all parameters (written at the top of the graphs)
- A file **state_model_2dof.mat** in your directory. This file contains the A, B, C, and D matrices for the state variable model of the system. If you subsequently type **load state_model_2dof** you will load these matrices into your workspace.

You need to be sure you have 4 points close to the resonant peaks of the transfer functions. This is particularly true if you have very small values of ζ (which correspond to very sharp peaks) At this point you probably should go back and add a few points near both the resonant peaks and nulls.

Your memo should include a description of each system (so you can set them up again), a table comparing the estimated values of the static gains, the natural frequencies, and the damping ratios using the two different methods (time domain and optimized frequency domain), and a brief comparison of the values. The damping ratios are often quite different, so that's OK. The other values should be close. You should include as attachments 7 graphs (3 log-decrement and 4 frequency response graphs), each with a figure number and caption. You should also include the data used for estimating the static gains.

Name _____

2 dof rectilinear systems (model 210)

Damper:	Yes No	
Left Spring:	stiff/light/none	Spring Number =
Number of Large Masse on First Cart:	2 3 4	
Number of Small Masses on First Cart:	1 2 3 4	
Middle Spring :	stiff/light	Spring Number =
Number of Large Masse on Second Cart:	2 3 4	
Number of Small Masses on Second Cart:	1 2 3 4	
Right Spring :	stiff/light	Spring Number =

state_model_2dof.mat is now named :

2 dof torsional systems (model 205)

Number of Masses on First Disk: 2 4

Each mass

a) is as far in (towards the center) as it can go

b) is as far out (away from the center) as it can go

c) is aligned so the outer edge is on the _____ ring from the outside

Number of Masses on Second Disk: 2 4

Each mass

a) is as far in (towards the center) as it can go

b) is as far out (away from the center) as it can go

c) is aligned so the outer edge is on the _____ ring from the outside