ECE-520 Lab 10

Modeling, Simulation, and Control of a 2 Degree of Freedom Inverted Pendulum System Using a Minimum Order Observer

<u>*Overview*</u> In this lab you will model, simulate, and control both a regular pendulum on a cart and an inverted pendulum on a cart. Note that the system we are controlling is unstable! The files you need will be in the **pendulum** folder, but you will need some of your minimum observer files. *You can work with a lab partner on this lab.*

For the two degree of freedom regular pendulum system we are going to control, with $q_1 = x$, $q_2 = \dot{x}$, $q_3 = \theta$, and $q_4 = \dot{\theta}$, we get the following state equations

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\frac{\omega_1^2}{\Delta}\right) & -\left(\frac{2\zeta_1\omega_1}{\Delta}\right) & \left(\frac{K_1\omega_1^2\omega_\theta^2}{\Delta}\right) & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{\omega_1^2\omega_\theta^2}{g\Delta}\right) & \left(\frac{2\zeta_1\omega_1\omega_\theta^2}{g\Delta}\right) & -\left(\frac{\omega_\theta^2}{\Delta}\right) & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\frac{K_2\omega_1^2}{\Delta}\right) \\ 0 \\ -\left(\frac{\omega_1^2\omega_\theta^2 K_2}{g\Delta}\right) \end{bmatrix} F$$

We need to identify all of these quantities to get the A and B matrices for the state variable description. For our system $D = 0$ and C is determined by whatever we want the output to be. Once we have the continuous time model we will sample it an use a minimum order controller to control the system. Once we can control the regular pendulum, we will determine the model for an inverted pendulum and then try and control that one using a minimum order observer.

<u>*Step 1: Set Up the System*</u>. Only the first cart should be able to move. In addition:

- There should be <u>*two large masses*</u> on the first cart.
- There should be <u>*no springs between the motor and the cart and one heavy spring between the first and second carts.*</u>
- <u>*Do not use a damper.*</u>
- The pendulum should be securely fastened to the first cart. It should rest on top of the masses and be securely tightened.
- The mass on the pendulum should be within about 2 or three inches from the pivot. Remember that the cart must be able to get under the center of mass of the pendulum in order to right it, so if the center of mass of the pendulum is too far away the cart will never be able to get under it. The ECP system should be moved to the edge of the bench, so that the pendulum is completely free to swing without hitting the bench.
- The wire to measure the position of the pendulum position encoder should be securely attached (with the screws) and the cart and pendulum should be free to move.

Step2a) Estimate of ω_θ

From the equations of motion, if we assume the cart is fixed, then $\ddot{x} = 0$ and we have

$$\ddot{\theta} + \omega_\theta^2 \theta = 0$$

This is the equation for a simple pendulum. If the pendulum is deflected a small angle and released, it will oscillate with frequency ω_θ.

To measure this:

- Set the input in **Model210_Openloop.mdl** to 0
- Set the X-Y graph in **Model210_Openloop.mdl** to measure the position of the pendulum. You may want to change the y-min and y-max values in the X-Y graph. We are measuring angles in radians, not degrees.
- Displace the pendulum and let it go. Since we are using a small angle assumption, the pendulum should not be displaced too far.
- Using Matlab, plot the displacement of the pendulum versus time, and determine the period of the pendulum, T_θ, and determine $\omega_\theta = \frac{2\pi}{T_\theta}$.

Step 2b) Estimation of ω_1 *and* ζ_1

If we assume there is not input ($F = 0$) and the pendulum does not move very much ($\ddot{\theta} \approx 0$) then we have

$$\frac{1}{\omega_1^2}\ddot{x} + \frac{2\zeta_1}{\omega_1}\dot{x} + x = 0$$

Use the log-decrement method to get estimates of ω_1 and ζ_1. Not that we don't really have the case of $\ddot{\theta} \approx 0$, but this approximation is not too far off. You will need to include these log-decrement results in your memo. ***Note that this is an equation in*** x***, not*** θ ***!!!***

Step 2c) Estimation of K_2

Applying a step input of amplitude A to the cart, estimate

$$K_2 = \frac{x_{ss}}{A}$$

Step 2d) Fitting the Estimated Frequency Response to the Measured Frequency Response

The transfer function between the input and the position of the first cart is given by

$$\frac{X(s)}{F(s)} = \frac{\omega_1^2 K_2 (s^2 + \omega_\theta^2)}{\left(1 - K_1 \omega_1^2 \frac{\omega_\theta^2}{g}\right) s^4 + \left(2\zeta_1 \omega_1\right) s^3 + (\omega_1^2 + \omega_\theta^2) s^2 + (2\zeta_1 \omega_1 \omega_\theta^2) s + \omega_1^2 \omega_\theta^2}$$

We will use this expression to determine K_1 and get better estimates of ω_1 and ζ_1, then we will have all of the parameters we need for our state variable model. We will be constructing the magnitude portion of the Bode plot and fitting this measured frequency response to the frequency response of the expected transfer function to determine these parameters. For each frequency $\omega = 2\pi f$ we have as input $F(t) = A\cos(\omega t)$ where, for out systems, A is measured in centimeters. After a transition period, the steady state output will be $x(t) = B\cos(\omega t + \theta)$ for the position of the first cart

Since we will be looking only at the magnitude portion of the Bode plot, we will ignore the phase angles.

You will go through the following steps

For frequencies $f = 0.5, 1, 1.5 ... 7.5$ Hz

- Make sure the first cart is free to move.
- Modify **Model210_Openloop.mdl** so the input is a sinusoid.
- Set the frequency and amplitude of the sinusoid. Try a small amplitude to start, like 0.1
- Compile **Model210_Openloop.mdl** *if necessary*. ***This is usually not necessary so only do it when you have to (the program will let you know.)***
- Connect **Model210_Openloop.mdl** to the ECP system. (The mode should be **External**.)
- Run **Model210_Openloop.mdl.** If the cart does not seem to move much, increase the amplitude of the input sinusoid. If the cart moves too much, decrease the amplitude of the input sinusoid. Note that if the cart hits the stops you will probably need to adjust the pendulum. *Be sure the system reaches steady state before you measure the amplitude!*
- Record the input frequency (f), the amplitude of the input (A), and the amplitude of the output (B) when the system is in steady state. You will probably want to use the file **get_B.m**.

You will probably notice that the output does not look quite as sinusoidal as usual. This is because we are not really giving the pendulum enough time to reach steady state. Enter the values of f, A, and B into the program **process_data_pendulum.m** (you need to edit the file)

At the Matlab prompt, type **data = process_data_pendulum**;

Step 3a) Modeling the Regular Pendulum

Run the program **model_pendulum_full.m.** There are 5 input arguments to this program:

- **data**, the measured data as determined by **process_data_pendulum.m**
- the estimated value of K_2
- ω_θ, the estimated frequency of the pendulum, in radians/sec
- ζ_1, the estimated damping ratio of the cart.
- ω_1, the estimated natural frequency of the cart, in radians/sec

The program **model_pendulum_full.m** will produce the following:

- A graph indicating the fit of the transfer function from the input to the position of the cart to the measured frequency response data.
- The optimal estimates of all parameters (written at the top of the graphs)
- A file **state_model.mat** in your directory**.** This file contains the A, B, C, and D matrices for the state variable model of the system. YOu should change the name of this file to **regular_pendulum.mat.**
- A list of the poles and zeros of the estimated transfer function. This allows you to see how close to a pole/zero cancellation you have.

You need to be sure you have 4 points close to the resonant peaks of the transfer functions. This is particularly true if you have very small values of ζ (which correspond to very sharp peaks.) In addition, you should add points near the frequency of the pole/zero cancellation to clean things up. You may also want to simulate the system at the natural frequency of the pendulum.

You should also compare the final estimates of the parameters with your initial estimates. The values for the all of the frequencies and gains should be fairly close to the final values. The damping ratios may be quite different.

Step 3b) Controlling the ECP System with Simulink for the Regular Pendulum

We want first of all to try and control a regular pendulum. This is actually a *regulator* problem in that we will be trying to maintain a set point (i.e., to keep the pendulum pointing down.)

a) Load the model file **regular_pendulum.mat** into your two degree of freedom minimum observer Matlab driver file.
b) Assume we are sampling with a sample interval Ts = 0.03 seconds (you can change this later if you need to).
c) The state vector for the original system will be in the order $\begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} & u_d \end{bmatrix}^T$, we need the system modified so the states are in the order $\begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} & u_d \end{bmatrix}^T$. You should also change the demux outputs in your Simulink models (both the model that will drive the ECP system and the model that we will use in our simulations.

d) You will need to change the variable **get_desired_states** so the ECP system knows the order we are using. It should be changed as follows:

```
get_desired_states = [1 0 0 0 0 0 0 0;
                      0 0 0 0 0 0 1 0;
                      0 1 0 0 0 0 0 0;
                      0 0 0 0 0 0 0 1];
```

e) Design a minimum order observer assuming both positions are known and simulate your system. Since this is a regulator problem there is no input into the system. Assume an initial displacement of 1 degree for the pendulum. Simulate your system. Your pendulum should come to rest within 1 second and your cart should not move more than 0.5 cm. You may have to change your poles around to make this happen.

f) Once your simulation is working properly, in Matlab's workspace set Tf=20 and compile your ECP driver (with the minimum order observer). Reset the system when the pendulum is pointing down and is at rest. Start the ECP system, and if you haven't screwed up the system should do nothing. Gently poke the pendulum with a ruler (in the cabinet, do not use your hands). The cart should move to keep the pendulum pointing down. Plot your estimated and true states for the system and show me the plot.

g) Once your regular pendulum is working correctly we can move onto the inverted pendulum. You need to run the file **model_inverted_pendulum_full.m,** which is exactly like **model_pendulum_full.m** except it models the inverted pendulum (all that really changes is the sign of some terms in the A and B matrices). This file produces a mathematical file **state_model.mat** which you should rename **inverted_pendulum.mat**.

h) Simulate your inverted pendulum as you did in part **d,** although your cart may move more than it did for the regular pendulum. Try and keep the cart from moving more than 1.2 cm from its initial position.

i) In your ECP driver file you will need to replace the yellow **ECP Model 210 Discrete Time** block with the **ECP Model 210 Discrete Time Inverted Pendulum** block. This block is available in the **inverted.mdl** file.

j) In Matlab's workspace set Tf = 20 and compile your ECP driver file (with the minimum order observer). Be sure the system is a rest and the pendulum is pointing down, then reset the system. Connect the ECP driver to the system and slowly raise the pendulum to 90 degrees. If you've done this before you know that the system is very sensitive to the initial displacement (from your simulations you should have seen that even being off by 1 degree can make it difficult to recover.) Once you think the pendulum is in the correct position, have your partner start your system. It is likely to take a few tries before it stays up by itself. *Be sure I see your pendulum working before you leave.*

There is no memo due, but you must get me or a designated representative to sign below, and turn this in at the end of lab:

Lab partners:____

I actually saw this think working: ______________________________________