ECE-520: Linear Control Systems
Homework 3

Due: Thursday December 15 at 5 PM

Problems

1) From Ogata, B-2-8.
Answer: $x(k)=5\delta(k)+4\delta(k-1)+3\delta(k-2)+2\delta(k-3)+\delta(k-4)$

2) From Ogata, B-2-9. Do only the partial fraction method
Answer: $x(k)=-8.333(0.5)^k u(k)+8.333(0.8)^k u(k)-2k(0.8)^{k-1}u(k)$

3) From Ogata, B-2-10. Use the integral method and check that your initial and final values for $x(k)$ are correct.
Answer: $x(0)=0, x(\infty)=0.37037, x(k)=[0.37037-1.48148(-0.8)^k+1.1111(-0.5)^k]u(k)$

4) From Ogata, B-2-11. Only do the integral method.
Answer: $x(k)=\delta(k)+2\delta(k-1)+u(k-2)$

5) From Ogata, B-2-13. Do this with both the integral method and partial fractions. Show that the answers are the same for the first few $x(k)$.
Answer: $x(k)=\delta(k)+1.2\delta(k-1)+[10-2.76(0.2)^{k-2}]u(k-2)$

6) From Ogata, B-2-14. Only the direct method.
Answer: $x(0)=0, x(1)=1, x(2)=0, x(3)=-3, x(4)=0, x(5)=5, x(6)=0, x(7)=-7, \ldots$

7) From Ogata, B-2-16.
Answers:
$$x(k)=[10+6.667(0.5)^k-16.667(0.8)^k]u(k)$$
$$x(k)=3.333[(0.8)^{k-1}-(0.5)^{k-1}]u(k-1)$$

8) From Ogata, B-2-17. Only analytically.
Answer: $x(k)=[4-(3+k)(0.5)^k]u(k)$

Preparation for Lab 3 (to be done individually)

9) In this problem we will examine the effects of a zero order hold, and see how we do various things in Matlab. Most of these have been done for you, but you need to look through the files and try and understand as much as you can.

a) From the class website, download the Matlab file **DT_driver.m** and the Simulink file **DT_openloop.mdl.** From your modeling, load the Matlab files containing your models for your 1 degree of freedom systems. (Be sure they are all in the same directory)

b) Modify **DT_driver.m** to load your rectilinear (210) model file. Simulate the continuous time and discrete time system for an input of a step with amplitude 0.1 cm for 2 seconds using a sampling time of 0.05 seconds. This is the default, so you should not have to change anything except loading your file. Look at how the continuous and discrete time signals look, and the effects of the zero order hold. Turn in your plot.

c) The *Nyquist sampling rate* is given as $T_s = \frac{1}{2 f_m}$ where T_s is the interval between samples and f_m is the highest frequency content of the signal we are sampling. Based on this criteria, with a sampling rate of $T_s = 0.05$ seconds, what is the highest allowed frequency in our input signal, f_m? (f_m is in Hz)

d) **Modify DT_driver.m** and **DT_openloop.mdl** so the input is a sinusoid with a frequency of 1 Hz (you need to convert this to radians) and an amplitude of 0.1 cm. Simulate the system for 5 seconds and look at the output. In order to see the effects of the zero order hold, you may have to change the plotting commands to plot (t+Ts/2,…) due to the way Simulink does the sine wave.(It won't be exact this way either, but it's a better approximation.) Turn in your plots.

e) Modify **DT_driver.m** and simulate your system for 5 seconds for input sine waves with amplitude 0.1 cm and various frequencies. What happens when the input frequency is that given by the Nyquist criteria? What is the highest frequency sine wave you think you can use and still get a *reasonably good representation* of the input? Plot your results with the highest acceptable frequency (there is no correct answer, only look in intervals of 0.5 radians/sec, i.e., 1.5 Hz, 2.0 Hz, 2.5 Hz, …)

10) Repeat problem 9 for your torsional system, except the input should be 1 degree (convert to radians) and change the output labels on the graphs. Your output should also be in degrees (or degrees/sec). Since the plant is in radians your will need to convert your output. Be sure to read in the correct model file for the torsional system.

We will be using a sampling rate of Ts = 0.05 seconds throughout this course. We could (and probably should) sample at a higher rate, but most of the frequency content of our systems can be fairly well represented with this sampling rate. Also, we can see what is happening better with a low sampling rate.