

**ECE-520: Discrete-Time Control Systems**  
Homework 10

Due: Thursday February 16 at 5 PM

*In this homework we will simulate and study optimal tracking for both one and two degree of freedom systems.*

*For the first four problems, utilize the continuous time torsional system models available from the class website, as well as the Simulink models **DT\_sv1\_optimal\_tracker.mdl**, **DT\_sv2\_optimal\_tracker.mdl**, and the Matlab files **tracker\_driver\_sv1.m**, **tracker\_driver\_sv2.m**, and **optimal\_tracker.m**. For all systems assume the sampling interval is  $T_s = 0.03$  seconds.*

1a) For the 1 degree of freedom torsional system, using time varying feedback gains and  $Q = 0.01$ ,  $R = 0.1$ , and  $Q_f = 0.05$ , for a 10 degree step input, you should get the output shown in Figure 1.

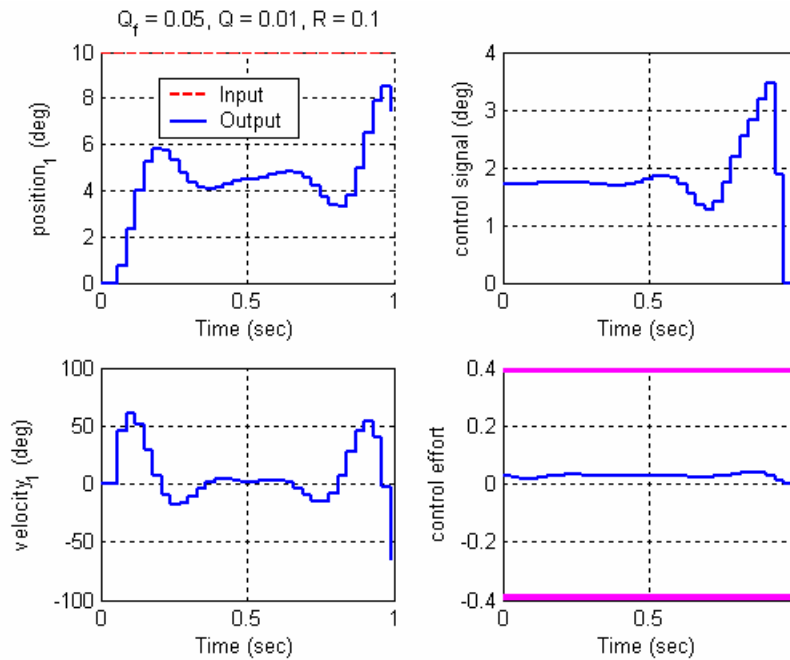


Figure 1. Results for Problem **1a**.

1b) Modify  $Q$ ,  $R$ , and  $Q_f$  so  $|r(N) - y(N)| < 0.5$  and the control effort does not saturate. Turn in your plot with your values of the weights you used.

1c) Modify  $Q$ ,  $R$ , and  $Q_f$  so  $|r(N) - y(N)| < 0.5$  and  $|r(k) - y(k)| < 0.4$  over 75% of the time. Turn in your plot with your values of the weights you used.

1d) Modify the program to use constant feedback gains (uncomment lines 88 and 89) and repeat problems 1b and 1c. Turn in your plots with your values of the weights you used.

1e) Change the signal we want to track to  $r(k) = 15^\circ \sin(10\pi k\Delta T)$ , modify  $Q$ ,  $R$ , and  $Q_f$  so the system tracks the reference signal reasonably well using constant feedback gains. You should note that the feedforward control signal leads the signal we want to track. Turn in your plots with your values of the weights you used.

2a) For the 2 degree of freedom torsional system, controlling the position of the **second** disk, using time varying feedback gains and  $Q = 0.01$ ,  $R = 0.1$ , and  $Q_f = 0.05$ , for a 10 degree step input, you should get the output shown in Figure 2.

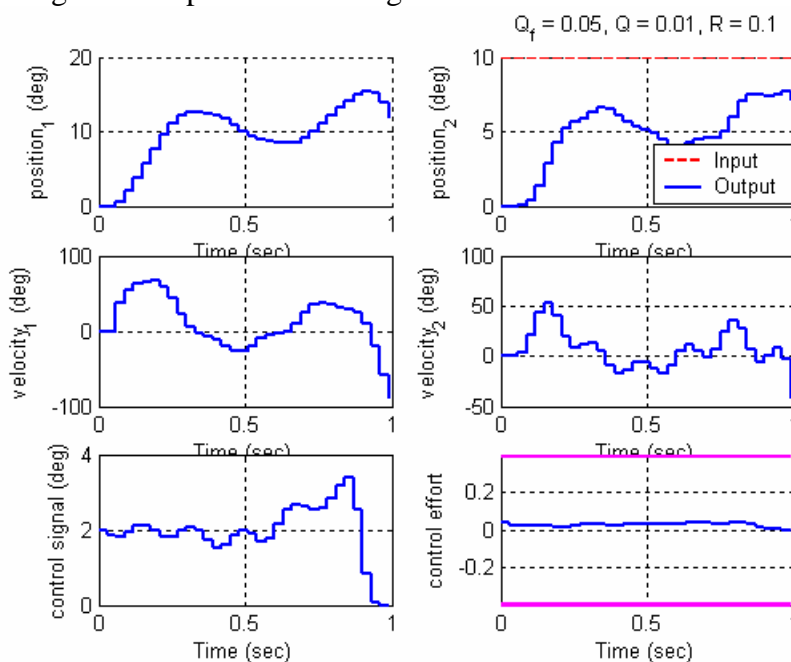


Figure 2. Results for Problem 2a.

2b) Modify  $Q$ ,  $R$ , and  $Q_f$  so  $|r(N) - y(N)| < 0.5$  and the control effort does not saturate. Turn in your plot with your values of the weights you used.

2c) Modify  $Q$ ,  $R$ , and  $Q_f$  so  $|r(N) - y(N)| < 0.5$  and  $|r(k) - y(k)| < 1$  over 75% of the time. Turn in your plot with your values of the weights you used.

2d) Modify the program to use constant feedback gains (uncomment lines 91 and 92) and repeat problems 1b and 1c. Turn in your plots with your values of the weights you used.

2e) Change the signal we want to track to  $r(k) = 15^\circ \sin(10\pi k\Delta T)$ , modify  $Q$ ,  $R$ , and  $Q_f$  so the system tracks the reference signal reasonably well using constant feedback gains. You should note that the feedforward control signal leads the signal we want to track. Turn in your plots with your values of the weights you used.

3a) For the 2 degree of freedom torsional system, controlling the position of the **first** disk, using time varying feedback gains and  $Q = 0.01$ ,  $R = 0.1$ , and  $Q_f = 0.05$ , for a 10 degree step input, you should get the output shown in Figure 3.

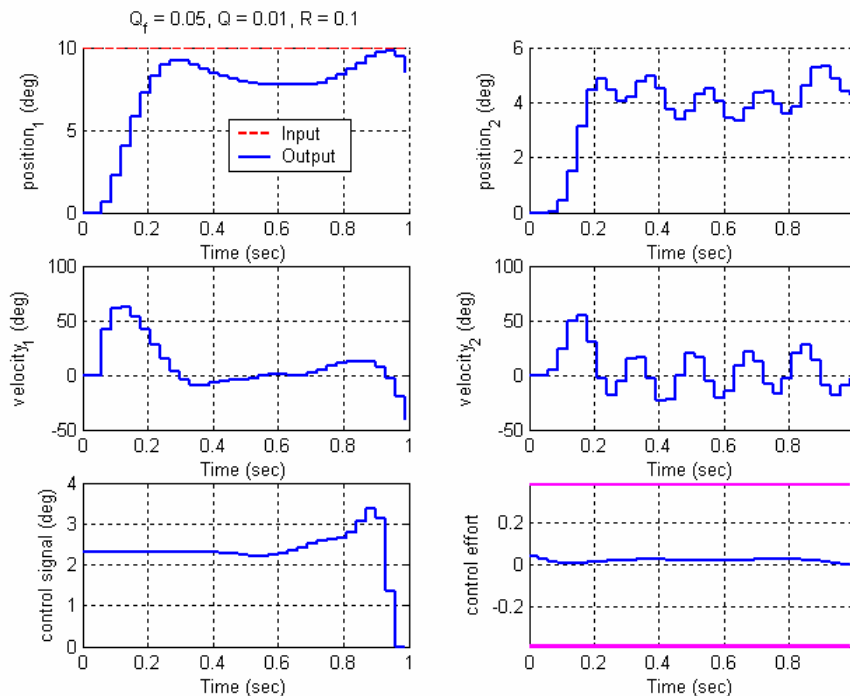


Figure 3. Results for Problem 3a.

3b) Modify  $Q$ ,  $R$ , and  $Q_f$  so  $|r(N) - y(N)| < 0.5$  and the control effort does not saturate. Turn in your plot with your values of the weights you used.

3c) Modify  $Q$ ,  $R$ , and  $Q_f$  so  $|r(N) - y(N)| < 0.5$  and  $|r(k) - y(k)| < 1$  over 75% of the time. Turn in your plot with your values of the weights you used.

3d) Modify the program to use constant feedback gains (uncomment lines 91 and 92) and repeat problems 1b and 1c. Turn in your plots with your values of the weights you used.

3e) Change the signal we want to track to  $r(k) = 15^\circ \sin(10\pi k\Delta T)$ , modify  $Q$ ,  $R$ , and  $Q_f$  so the system tracks the reference signal reasonably well using constant feedback gains. You

should note that the feedforward control signal leads the signal we want to track. Turn in your plots with your values of the weights you used.

4) The class website contains the file **regular\_pendulum.mat**, which is a continuous time model for a pendulum on a cart. There are four states in this model, and the state vector is

$\begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}^T$ , where  $x$  is the position of the cart, and  $\theta$  is the position of the pendulum.

- a) Load the continuous time model for the pendulum into your program that constructs the minimum order observer for a two degree of freedom discrete-time system. (Assume there is a delay, which is what we usually assume.) Assume we are sampling with a sampling interval of 0.03 seconds.
- b) Also assume that the known states are the positions, and the states to be estimated by a minimum order observer are the velocities. Hence you need to modify your system so the state vector is now  $\begin{bmatrix} x & \theta & \dot{x} & \dot{\theta} \end{bmatrix}^T$
- c) Assume the cart is initially at rest and the pendulum is initially at 1 degree (with zero velocity). The minimum order estimator assumes the system is initially at rest. There is no input into the system, so set either the amplitude or Gpf equal to zero.
- d) Find state feedback and observer poles so the system returns to rest within 1 second, the cart does not move more than 0.5 cm in either direction, and the motor does not saturate. Simulate your system, turn in your plot and be sure to include your pole locations.
- e) Download the file **inverted\_pendulum.mat** from the class web site. This file contains a continuous time model for the same pendulum in an inverted configuration. Load this program into your Matlab simulation, and repeat part **d**. However, your cart is allowed to move with 1.2 cm in either direction this time. Simulate your system, turn in your plot and be sure to include your pole locations