ECE-520: Discrete-Time Control Systems
Homework 4

Due: Thursday September 30 in class

1) For this problem, assume $\underline{p} = \begin{bmatrix} a \\ b \end{bmatrix}, \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ and show the following:

a) for $f(\underline{x}) = \underline{p}^T \underline{x}, \frac{df}{d\underline{x}} = \underline{p}^T$

b) for $f(\underline{x}) = \underline{x}^T \underline{p}, \frac{df}{d\underline{x}} = \underline{p}^T$

c) for $f(\underline{x}) = A\underline{x}, \frac{df}{d\underline{x}} = A$

d) for $f(\underline{x}) = A^T \underline{x}, \frac{df}{d\underline{x}} = A^T$

e) for $f(\underline{x}) = \underline{x}^T A\underline{x}, \frac{df}{d\underline{x}} = x^T(A + A^T)$

2) The error vector $\underline{e}$ between observation vector $\underline{d}$ and the estimate of the input $\hat{\underline{x}}$ is $\underline{e} = \underline{d} - A\hat{\underline{x}}$. We want to weight the errors by a *symmetric* matrix R. Find $\hat{\underline{x}}$ to minimize $\underline{e}^T \mathrm{R}\, \underline{e}$. (This is a weighted least squares.)

3) Assume we expect a process to follow the equation $y(t) = \frac{1}{ct + d\sqrt{t}}$, and we have measurements:

t	1.0	2.0	3.0	4.0	5.0	6.0
$y(t)$	0.30	0.21	0.14	0.12	0.11	0.09

a) Determine a least squares estimate of the parameters c and d.

b) Using your least squares estimate of the parameters, estimate the value of $y(2.5)$.

c) Assume we want to use a weighted least squares with r = diag([2 2 2 1 1 1]). Determine the weighted least squares estimate of parameters c and d.

4) Assume we expect a process to follow the following equation: $\gamma(x) = \varepsilon e^{\beta x}$

Assume we measure $\gamma(x)$ at various locations x:

x	0.0	0.1	0.4	2.0	4.0
$\gamma(x)$	2.45	2.38	2.30	1.40	0.70

a) Determine a least squares fit to the parameters ε and β. (*Hint: Try logarithms.*)

b) Estimate $\gamma(3.0)$

5) Assume we have an experimental process we are modeling, and, based on sound physical principles, we believe the relationship between x and y to be

$$y(x) = \left(\frac{\alpha}{x}\right)^{\beta}$$

and we have the following measurements:

x	1.0	2.0	3.0	4.0
$y(x)$	8.0	1.0	0.1	0.1

a) Find a least squares estimate for α and β. (Hint: You cannot solve for α directly. Let $w = \beta \log(\alpha)$, solve for w and β, and then infer α.)

b) Estimate $y(3.5)$.