ECE-520: Discrete-Time Control Systems
Homework 3

Due: Thursday September 23 in class
Exam 1, Friday September 24

1) For each of the following transfer functions, determine if the system is asymptotically stable, and if so, the estimated 2% settling time for the system. Assume the sampling interval is $T = 0.1$ seconds.

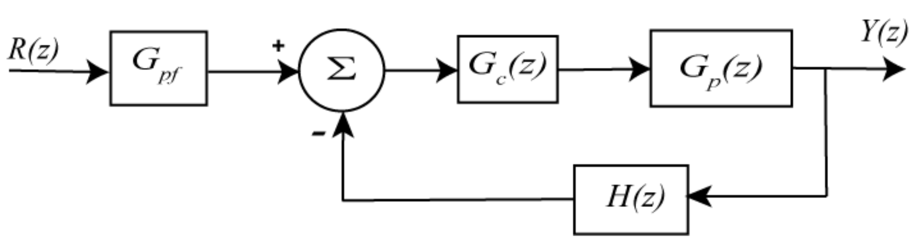
a) $H(z) = \dfrac{z+2}{(z-0.1)(z+0.2)}$ **d)** $H(z) = \dfrac{1}{z^2 + z + 0.5}$

b) $H(z) = \dfrac{1}{(z-2)(z+0.5)}$ **e)** $H(z) = \dfrac{z-1}{z^2 + 0.5z + 0.2}$

c) $H(z) = \dfrac{1}{(z-0.1)(z-0.5)}$ **f)** $H(z) = \dfrac{1}{z^2 + z + 5}$

Scambled Answers: 0.497, 0.58 , 1.15, 0.24, two unstable systems

2) For the following system, assuming the closed loop systems are stable, determine the prefilter gain G_{pf} that will result in zero steady state error for a unit step input. Are any of these systems type one systems?



a) $G_p(z) = \dfrac{0.2}{z^2 + 0.1z + 0.2}$, $G_c(z) = \dfrac{z}{z-1}$, $H(z) = 1$

b) $G_p(z) = \dfrac{0.2}{z^2 + 0.1z + 0.2}$, $G_c(z) = \dfrac{1}{z}$, $H(z) = 1$

c) $G_p(z) = \dfrac{1}{z^2 + 0.4z + 0.04}$, $G_c(z) = \dfrac{0.2}{z+0.2}$, $H(z) = \dfrac{1}{z+0.2}$

Answers: 7.5, 9.47, one is type one (so the prefilter has value 1)

3) Consider the continuous-time plant with transfer function

$$G_p(s) = \frac{1}{(s+1)(s+2)}$$

We want to determine the discrete-time equivalent to this plant, $G_p(z)$, by assuming a zero order hold is placed before the continuous-time plant to convert the discrete-time control signal to a continuous time control signal.

Show that if we assume a sampling interval of T, the equivalent discrete-time plant is

$$G_p(z) = \frac{z(0.5 - e^{-T} + 0.5\mathrm{e}^{-2T}) + (0.5e^{-T} - e^{-2T} + 0.5e^{-3T})}{(z - e^{-T})(z - e^{-2T})}$$

Note that we have poles were we expect them to be, but we have introduced a zero in going from the continuous time system to the discrete-time system.

4) In this problem assume the feedback configuration shown in Problem 2.

a) Assume

$$H(z) = z^{-1},\ G_p(z) = b_0^p,\ G_c(z) = \frac{b_0^c z + b_1^c}{z-1}$$

If the plant is equal to 3 and we want all of the closed loop poles at -0.5, then the controller and closed loop transfer functions are

$$G_c(z) = \frac{\frac{2}{3}z + \frac{1}{12}}{z-1},\ G_o(z) = \frac{z(2z+0.25)}{z^2+z+0.25}$$

b) Assume

$$H(z) = 1, G_c(z) = \frac{b_0^c z + \mathrm{b}_1^c}{z-1}, G_p(z) = \frac{b_0^p}{z + z_1^P}$$

If the plant is equal to $\frac{2}{z-0.5}$ and the closed loop poles are at -0.5 and -0.333, then the controller and the closed loop transfer function are

$$G_c(z) = \frac{1.167z - 0.167}{z-1}, G_o(z) = \frac{2.333z - 0.333}{z^2 + 0.833z + 0.167}$$