ECE-520: Discrete-Time Control Systems
Homework 2

Due: Thursday September 16 at the beginning of class

1) For impulse response $h(n)=\left(\frac{1}{5}\right)^{n}u(n)$ and input $x(n)=u(-n)$, show that the system output is $y(n)=\frac{5}{4}u(-n)+\frac{1}{4}\left(\frac{1}{5}\right)^{n-1}u(n-1)$ by

a) evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}x(n-k)h(k)$

b) evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$

2) For impulse response $h(n)=\left(\frac{1}{3}\right)^{n-2}u(n-3)$ and input $x(n)=\left(\frac{1}{2}\right)^{-n}u(2-n)$, show that the system output is $y(n)=\frac{1}{5}2^{n-2}u(5-n)+\frac{8}{5}\left(\frac{1}{3}\right)^{n-5}u(n-6)$ by

a) evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}x(n-k)h(k)$

b) evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$

3) For the z-transform

$$X(z)=\frac{3}{z-2}$$

a) Show that, by multiplying and dividing by z and then using partial fractions, the corresponding discrete-time sequence is

$$x(k)=-\frac{3}{2}\delta(k)+\frac{3}{2}2^{k}u(k)$$

b) By starting with the z-transform

$$Y(z)=\frac{3z}{z-2}$$

and the z-transform properties, show that

$$x(k)=3\,2^{k-1}u(k-1)$$

4) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n-3}u(n-1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n+1}u(n-2)$, use z-transforms of the input and impulse response to show the system output is $y(n)=\left[\left(\frac{1}{2}\right)^{n}-\left(\frac{1}{4}\right)^{n-1}\right]u(n-3)$

5) For $h(n)=\left(\frac{1}{a}\right)^{n}u(-n)+a^{n}u(n)$ with $|a|<1$,using the two-sided z transform to show that

$$H(z)=\frac{1}{1-az}+\frac{1}{1-az^{-1}}$$

and the region of convergence is $|a|<|z|<\frac{1}{|a|}$

6) In class we derived the relation ships

$$G(z)=\frac{rz[z\cos(\theta)-\gamma\cos(\beta-\theta)]}{z^2-2\gamma\cos(\beta)z+\gamma^2}=\frac{Az^2+Bz}{z^2+2Cz+\gamma^2}\leftrightarrow g(n)=r\gamma^n\cos(\beta n+\theta)u(n)$$

with

$$\beta=\cos^{-1}\left(\frac{-C}{\gamma}\right),\theta=\tan^{-1}\left(\frac{CA-B}{A\sqrt{\gamma^2-C^2}}\right),r=\sqrt{\frac{A^2\gamma^2+B^2-2ABC}{\gamma^2-C^2}}$$

a) Us the above formulas to find the impulse response $g(n)$ for $G(z)=\frac{z^2+0.5z}{z^2+0.2z+0.125}$. **b)** Compute $g(n)$ from part **a** for $n=0,1,2,3,4$ and then perform long division to verify that your answer to **a** is correct for these terms

c) Determine the unit step response $y(n)$ for $G(z)=\frac{1}{z^2+0.1z+4}$, by using the form

$$\frac{Y(z)}{z}=\frac{1}{(z-1)(z^2+0.1z+4)}=\frac{\alpha_1}{z-1}+\frac{\alpha_2 z+\alpha_3}{z^2+0.1z+4}$$

Hint: α_1 can be found using the cover-up method, α_2 can be found by multiplying both sides by z and letting $z\to\infty$, and α_3 can be found by substituting a convenient value for z, like $z=0$.

d) Compute $y(n)$ from part **e** for $n=0,1,2,3,4$ and then perform long division to verify that your answer to **e** is correct for these terms

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7) Consider the following difference equation

$$x(k+2) - 4x(k+1) + 4x(k) = f(k)$$

Assume all initial conditions are zero.

a) Determine the *impulse response* of the system, i.e., the response $x(k)$ when $f(k) = \delta(k)$.

b) Determine $x(0), x(1), x(2), x(3),$ and $x(4)$ from your answer to **a.** Compare this answer with the known values of $x(0)$ and $x(1)$. Using the difference equation compute $x(3)$ and $x(4)$ and compare these values to those in your solution.

c) Determine the *step response* of the system, i.e., the response $x(k)$ when $f(k) = u(k)$

d) Determine $x(0), x(1), x(2), x(3),$ and $x(4)$ from your answer to **c.** Compare this answer with the known values of $x(0)$ and $x(1)$. Using the difference equation compute $x(3)$ and $x(4)$ and compare these values to those in your solution.

8) Consider the difference equation

$$x(k+2) - 5x(k+1) + 6x(k) = f(k)$$

where $f(k) = u(k)$, a unit step. Assume $x(0) = 1$ and $x(1) = 1$.

a) Determine the *Zero Input Response* (ZIR), $x_{ZIR}(k)$. This is the part of the solution $x(k)$ due to the initial conditions alone (assume the input is zero).

b) Determine the *Zero State Response* (ZSR), $x_{ZSR}(k)$. This is the part of the solution $x(k)$ due to the input alone (assume all initial conditions are zero).

c) Find the total response $x(k) = x_{ZIR}(k) + x_{ZSR}(k)$

d) Find the transfer function and the impulse response.

e) Determine $x(0), x(1), x(2), x(3),$ and $x(4)$ from your answer to **c.** Compare this answer with the known values of $x(0)$ and $x(1)$. Using the difference equation compute $x(3)$ and $x(4)$ and compare these values to those in your solution.