ECE-497-3: Inverse Problems in Engineering
Homework #7

Due: Friday Feb 21, 2003

1) Implement the Generalized Cross Validation method of choosing b for the dynamic programming method for zero order regularization. Be sure your method works for both the case of known initial state, and for the case of unknown initial state. Using the single input/single output state variable models from homework 4, and your implementation for the generalized cross validation from homework 6, compare the estimated values of b using the brute force and dynamic programming methods (for **zero order** only). For the same noise realization, you should get very similar values of b. Be sure to demonstrate this in your write-up.

2) Implement the inverse heat transfer example from Section 4.2 in the book. Use only the dynamic programming method for the solution, and use the GCV method for choosing the value of b. Assume the time step is $h = 0.05$ (or smaller) and use at least 30 sample (input/observation) points. Use σ of both 0.25 and 1.0, where values should be

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d_{measured} = d_{true} + sigma*randn(length(d),1)
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Try assuming the initial temperature is known, and then assuming the initial temperature is unknown. Do not expect to get the exact results from the book, but it should be close. (Assume the initial temperature in the bar is a constant, such as 0 everywhere.)

You will need to use the following form of the Crank-Nicholson formula. For the continuous time state variable equations

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{b}g(t)$$

the discrete-time equations are

$$\mathbf{x}_{k+1} = (I - A\frac{h}{2})^{-1}(I + A\frac{h}{2})\mathbf{x}_k + h(I - A\frac{h}{2})^{-1}\mathbf{b}\ g_k$$

3) Using the model and parameters from the inverse , implement zero, first, and second order regularization using the dynamic programming and GCV to select the b parameter. Try this both assuming the initial temperature is known, and assuming the initial temperature is unknown.