ECE 497-3: Inverse Problems in Engineering

Homework #1

Due: Friday December 13, 2002

For problems 1-5, let

$$\underline{a} = \begin{bmatrix} a \\ b \end{bmatrix}, \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

and show the following:

1) for $f(\underline{x}) = \underline{a}^T \underline{x}$, $\frac{df}{d\underline{x}} = \underline{a}$

2) for $f(\underline{x}) = \underline{x}^T \underline{a}, \frac{df}{d\underline{x}} = \underline{a}$

3) for $f(\underline{x}) = A\underline{x}, \frac{df}{d\underline{x}} = A^T$

4) for $f(\underline{x}) = A^T \underline{x}, \frac{df}{d\underline{x}} = A$

5) for $f(\underline{x}) = \underline{x}^T A \underline{x}, \frac{df}{d\underline{x}} = (A + A^T)\underline{x}$

6) The error vector $\underline{e}$ between observation vector $\underline{d}$ and estimate of the input $\hat{\underline{x}}$ is $\underline{e} = \underline{d} - A\hat{\underline{x}}$. We want to weight the errors by a matrix R, where R is symmetric ($R = R^T$). Find $\hat{\underline{x}}$ to minimize $\underline{e}^T R \underline{e}$. (This is a weighted least squares.)

7) Show that any matrix A can be written as the sum of a symmetric matrix and a skew symmetric matrix. That is,

$$\begin{aligned} A &= R + Q \\ R &= R^T \\ Q &= -Q^T \end{aligned}$$

Determine R and Q.

8) Assume we expect a process to follow the following equation

$$y(t) \quad = \quad \frac{1}{ct + d\sqrt{t}}$$

Assume we measure the $y(t)$ at various times t:

t	$y(t)$
1.0	0.30
2.0	0.21
3.0	0.14
4.0	0.12
5.0	0.11
6.0	0.09

a) Determine a least squares estimate of the parameters c and d.
b) Estimate the value of $y(t)$ at $t = 2.5$.
c) Suppose we believe all measurements made before time $t = 3.5$ are twice as reliable as those made later. Determined a reasonable weighted least squares estimated of c and d.

9) Assume we expect a process to follow the following equation

$$\gamma(x) = \epsilon e^{\beta x}$$

Assume we measure the $\gamma(x)$ at various locations x:

x	$\gamma(x)$
0.0	2.45
0.1	2.38
0.4	2.30
2.0	1.40
4.0	0.70

a) Determine a least squares estimate of the parameters ϵ and β. (*Hint: Try logarythms...*)
b) Estimate the value of $\gamma(x)$ at $x = 3.0$.