ECE-420: Discrete-Time Control Systems
Homework 3

Due: **Friday** September 25 at the beginning of class

1) Consider the continuous-time plant with transfer function

$$G_p(s) = \frac{1}{(s+1)(s+2)}$$

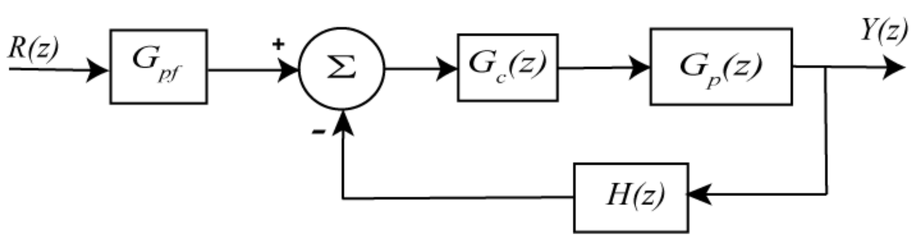
We want to determine the discrete-time equivalent to this plant, $G_p(z)$, by assuming a zero order hold is placed before the continuous-time plant to convert the discrete-time control signal to a continuous time control signal.

Show that if we assume a sampling interval of T, the equivalent discrete-time plant is

$$G_p(z) = \frac{z(0.5 - e^{-T} + 0.5\mathrm{e}^{-2T}) + (0.5e^{-T} - e^{-2T} + 0.5e^{-3T})}{(z - e^{-T})(z - e^{-2T})}$$

Note that we have poles were we expect them to be, but we have introduced a zero in going from the continuous time system to the discrete-time system.

2) In this problem assume the feedback configuration shown below.



a) Assume

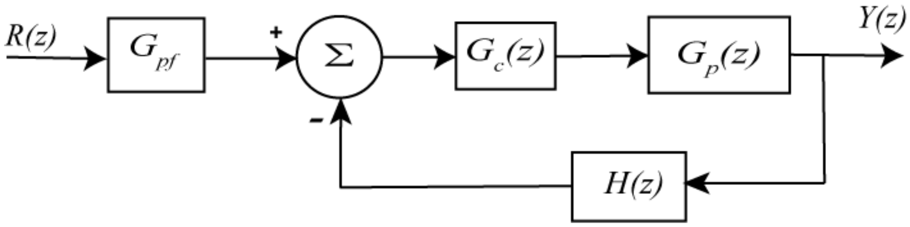
$$H(z) = z^{-1},\ G_p(z) = b_0^p,\ G_c(z) = \frac{b_0^c z + b_1^c}{z-1}$$

If the plant is equal to 3 and we want all of the closed loop poles at -0.5, show that the controller and closed loop transfer functions are $G_c(z) = \dfrac{\frac{2}{3}z + \frac{1}{12}}{z-1},\ G_o(z) = \dfrac{z(2z+0.25)}{z^2+z+0.25}$

b) Assume $H(z) = 1, G_c(z) = \dfrac{b_0^c z + \mathrm{b}_1^c}{z-1}, G_p(z) = \dfrac{b_0^p}{z + a_1^P}$ If the plant is equal to $\dfrac{2}{z-0.5}$ and the closed loop poles are at -0.5 and -0.333, show that the controller and the closed loop transfer function are

$$G_c(z) = \frac{1.167z - 0.167}{z-1},\ G_o(z) = \frac{2.333z - 0.333}{z^2 + 0.833z + 0.167}$$

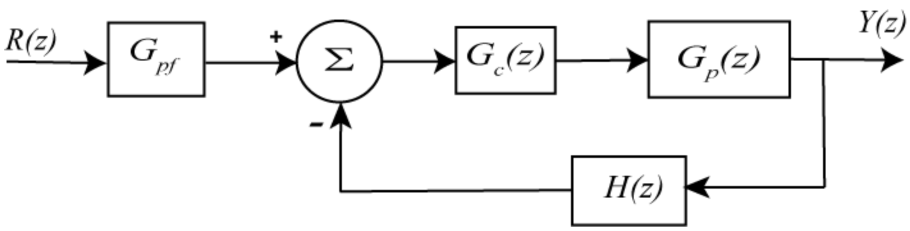
3) Assume the following feedback configuration



If $H(z) = z^{-1}, G_c(z) = \dfrac{c(z+a)}{z+b}, G_p(z) = \dfrac{2}{z+1}$ determine the parameters *a*, *b*, and *c* so all of the closed loop poles are at 0.5.

Hint: $(z-0.5)^3 = z^3 - 1.5z^2 + 0.75\,z - 0.125$

4) Assume the following feedback configuration



If $H(z) = z^{-1}, G_c(z) = \dfrac{c(z+a)}{(z-1)(z+b)}, G_p(z) = \dfrac{2z}{z+1}$ determine the parameters *a*, *b*, and *c* is the desired closed loop poles are roots of the equation $\Delta(z) = z^3 + d_1 z^2 + d_2 z + d_3$

5) Prove or disprove the following claims: if $u, v,$ and w are linearly independent vectors, then so are

a) $u, u+v, u+v+w$
b) $u+2v-w, u-2v-w, 4v$
c) $u-v, v-w, w-u$
d) $-u+v+w, u-v+w, -u+v-w$

Note: You must do this for arbitrary vectors. ***<u>Do Not</u>*** assume $u, v,$ and w are specific vectors.

6) Determine the ranks of the following matrices. Do not use a calculator or computer, look for linearly independent columns or rows.

a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}$ d) $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 3 & 1 & 2 \end{bmatrix}$

e) $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ f) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Answers (scrambled): 3, 2, 2, 1, 2, 2

7) For the following matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

a) Find a set of vectors that form a basis for the null space of A.

b) Is the vector $\underline{n} = \begin{bmatrix} 2 & 2 & -2 & 2 \end{bmatrix}^T$ in the null space of A? That is, can you represent this vector as a linear combination of your basis vectors?

8) For the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

a) Find a set of vectors that form a basis for the null space of A.

b) Is the vector $\underline{n} = \begin{bmatrix} 2 & 6 & -2 & -1 \end{bmatrix}^T$ in the null space of A? That is, can you represent this vector as a linear combination of your basis vectors?

9) For the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & 3 & 5 \end{bmatrix}$$

a) Find the rank of A (the number of linearly independent rows or columns).

b) Determine two vectors that span the null space of A.