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ECE-420
Exam 2

Fall 2015

No Calculators allowed. You must show your work to receive credit.

Problem 1 __________/50

Problem 2 __________/20

Problem 3 __________/10

Problem 4 __________/20

Total _______________

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1) Consider the discrete-time state variable model

$$\underline{x}(k+1) = G\underline{x}(k) + Hu(k)$$

with the initial state $x(0) = 0$. Let

$$G = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0$$

a) Using the Cayley-Hamilton Theorem, show that we can then write

$$x(3) = \begin{bmatrix} GH & H \end{bmatrix} \tilde{u}(2)$$

and determine $\tilde{u}(2)$

b) Use the direct eigenvalue assignment method ($\begin{bmatrix} \lambda I - G & | & H \end{bmatrix}$) to determine k_1 and k_2 so the closed loop poles are at -1 and -2.

c) Use Ackermann's method to determine k_1 and k_2 so the closed loop poles are at -1 and -2.

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2) We have been assuming the following form for our discrete-time transfer functions with input $R(z)$ and output $Y(z)$

$$G_p(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Cross multiplying we get

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) = b_0 U(z) + b_1 z^{-1} U(z) + b_2 z^{-2} U(z)$$

In the time-domain this becomes

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_0 u(n) + b_1 u(n-1) + b_2 u(n-2)$$

Assume we have the following time-varying plant

$$G_p(z) = \frac{0.5z^{-1} + 1z^{-2}}{1 + 0.5z^{-1} + 0.1z^{-2}} \quad t < 2$$

$$G_p(z) = \frac{2z^{-1} + 1z^{-2}}{1 - 0.2z^{-1} + 0.3z^{-2}} \quad 2 < t < 4$$

$$G_p(z) = \frac{1z^{-1} - 1z^{-2}}{1 + 0z^{-1} - 0.3z^{-2}} \quad t > 4$$

We utilize a recursive least squares algorithm and get the results shown in the graph on the next page. Is the algorithm working or not? Explain you answer. If the algorithm is failing indicate all the ways you feel the algorithm is failing. If the algorithm is working, explain how the performance might be improved.

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3a) Explain why a recursive least squares algorithm is not likely going to work well to determine a transfer function if the input to the system is a sinusoid.

3b) Assume the forgetting factor is $\lambda = 0.5$. Explain what you expect to observe as λ is ***increased*** significantly.

3c) Assume the forgetting factor is $\lambda = 0.5$. Explain what you expect to observe as λ is ***decreased*** significantly.

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4) Consider the continuous-time state variable model with a delay longer than one sample interval,

$$\dot{x}(t) = Ax(t) + Bu(t - T - \tau) \quad 0 < \tau < T$$
$$y(t) = Cx(t) + Du(t)$$

The solution for the continuous-time state equation is

$$x(t) = e^{A(t-t_o)}x(t_o) + \int_{t_o}^{t} e^{A(t-\lambda)}Bu(\lambda - T - \tau)d\lambda$$

Assume $t_o = nT$, $t = (n+1)T$, and $u(t) = u(nT)$ for $nT \le t < (n+1)T$

Show that the discrete-time state variable model for this system can be written as

$$\begin{bmatrix} x(n+1) \\ u(n-1) \\ u(n) \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(n) \\ ? \\ ? \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} u(n)$$

Specifically, determine expressions for all 5 of the ?. You do not need to simplify any of the integrals, but you need to write them out.

Hint: Draw a sketch of u(nT) for various times, such as u((n-2)T), u((n-1)T), u(nT), u(n+1)T), etc. like we did in class.

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