Name____ Mail Box __________________

ECE-420
Exam 1

Fall 2015

<u>Calculators and Laptops cannot be used</u>. You must show your work to receive credit.

Problem 1 __________/10

Problem 2 __________/15

Problem 3 __________/12

Problem 4 __________/15

Problem 5 __________/8

Problem 6 __________/20

Problem 7 __________/20

Total _______________

Name____ Mail Box ____________________

1a) Use long division to determine the first three nonzero terms in **<u>the impulse response</u>** for the following transfer function $H(z) = \frac{z+1}{z^2+z+1}$

1b) Assume a system has impulse response $h(n) = (0.5)^n u(n)$ Determine the output if the input is $x(n) = 2\delta(n) - 3\delta(n-2)$

Name____ Mail Box __________________

2) Assume we have a process which we expect to be able to model as $g(t) = \alpha t + \beta$. We perform an experiment and determine values for $g(t)$ for a number of different values of t, so we have the values

$$
\begin{array}{ll}
t_0, & g(t_0) \\
t_1, & g(t_1) \\
t_2, & g(t_2) \\
t_3, & g(t_3) \\
t_4, & g(t_4)
\end{array}
$$

We want to estimate α and β using a least squares type of approach, with the model $z = Aw$. Describe how you would do this. Be sure in your description, to indicate what the z, w, and A are in terms of the values given $t_i, g(t_i)$

Name____ Mail Box ___________________

3) Determine the rank of each of the following matrices (just write the number):

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

Name____ Mail Box ___________________

4) Consider the discrete-time state variable system,

$$x(n+1)=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}x(n)+\begin{bmatrix}1\\ 0\end{bmatrix}u(n)$$

Assuming we start at the origin (the point 0,0) after 3 time steps we have

$$x(3)=\begin{bmatrix}1 & 1 & 1\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}u(0)\\ u(1)\\ u(2)\end{bmatrix}=Au$$

a) If the input is $u^T=\begin{bmatrix}1 & 1 & 1\end{bmatrix}$ determine $x(3)$

b) Find a **unit vector** in the null space of the matrix A

c) Is the input vector in step (a) the input with the smallest magnitude to go from the origin to the point $x(3)$? Explain your answer.

Name____ Mail Box ___________________

5) Use a Lagrange multiplier to find the point on the plane $2x + 4y - 6 = 0$ nearest the origin

Name____ Mail Box ___________________

6) Consider the overdetermined system $Ax = b$. Rather than just performing a normal least squares solution, we want to penalize the magnitude of the solution by some weighting scalar μ. Thus we want to find the value of x that minimizes

$$H = \| Ax - b \|^2 + \mu \| x \|^2 = (Ax - b)^T (Ax - b) + \mu (x^T I x)$$

a) Show that the solution to the above minimization problem is given by $x = (A^T A + \mu I)^{-1} A^T b$

b) Assume now we compute the singular value decomposition of A, $A = USV^T$. Recall that U and V are unitary matrices, so $U^T U = V^T V = I$. This means that $U^{-1} = U^T$ and $V^{-1} = V^T$. S is a diagonal matrix with elements σ_i on the diagonal (assume *S* has full rank and is invertible). Assume that we can write $b = U\beta$ and $x = V\alpha$. Determine how α and β are related. *Hint: it may be helpful to write* $I = VIV^T$

Hints: μ *is* ***<u>not</u>*** *a Lagrange multiplier, there are no Lagrange multipliers in this problem. You can do both parts of this problem independently.*

Cultural Point: This technique is known as zero order Tikhonov regularization.

Name____ Mail Box ___________________

7) For impulse response $h(n) = \left(\frac{1}{2}\right)^{n} u(n-2)$ and input $x(n) = \left(\frac{1}{2}\right)^{-n} u(-\mathrm{n}+1)$, the system output can be written as $A(n)u(n-3) + B(n)u(2-n)$. Determine an expression for **both** $A(n)$ **and** $B(n)$. You do not need to simplify your expression but you must evaluate all sums. For my sanity (not that you care), evaluate the convolution using the form $y(n) = \sum_{k=-\infty}^{k=\infty} h(n-k)x(k)$

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