

ECE-420: Discrete-Time Control Systems
Homework 8

Due: Friday November 7 at the beginning of class

In this homework we will simulate and study the prediction full order observer coupled with state variable feedback, and then we will add on integral control. As you hopefully remember, having an integrator in our system is generally preferable to using a prefilter for achieving a zero steady state error for a step input, since it will help overcome errors in the plant model.

Some of the things we will do in this homework are a bit weird, but they are being done for the sake of demonstrating that your programs are working. Specifically, having the initial estimates of the states so large is quite unrealistic, but it does allow us to see if the estimated states are converging to the true states. However, these initial large states force us to utilize very small closed loop poles for the state feedback and the integrators. Strange as it may seem, we need to use these small poles to keep the system from overwhelming the motor (having the control effort reach saturation).

For all of these problems, utilize the torsional system models available from the class website that you used last week. This system should be sampled with $T_s = 0.05$.

(1 dof Integral Control)

1a) Copy your program **DT_sv1_driver.m** to a new file **DT_sv1_servo_driver.m**, then copy the Simulink file **DT_sv1.mdl** to **DT_sv1_servo.mdl** . Modify these new files to implement both state variable feedback and an integrator control . Your programs should be able to use either **place** or **acker** to place the closed loop poles directly. Note that if our system has an integrator in it, there should be no prefilter.

b) Modify the Matlab code (the poles and the **place** command) to place the poles of the closed loop system in such a way so that for a 15 degree step input, for the first disk:

- the settling time for your system is less than 1 s
- the percent overshoot for your system is less than 20%
- the control effort does not hit a limiter (does not saturate)
- the steady state error is zero

Simulate the system and turn in your graph.

(2 dof Integral Control)

2a) Copy your program **DT_sv1_servo_driver.m** to a new file **DT_sv2_servo_driver.m**, then copy the Simulink file **DT_sv1_servo.mdl** to **DT_sv2_servo.mdl** . Modify these new files to implement both state variable feedback and an integrator control. Your programs should be able to use either **place** or **acker** to place the closed loop poles directly. Note that if our system has an integrator in it, there should be no prefilter. Modify the Matlab code (the poles and the **place** command) to place the poles of the closed loop system in such a way so that for a 15 degree step input, for the first disk:

- the settling time for your system is less than 1 s
- the percent overshoot for your system is less than 20%

- the control effort does not hit a limiter (does not saturate)
- the steady state error is zero

Simulate the system *and turn in your graph*.

2b) Repeat part a to control the position of the second disk. *Turn in your graph*.

(1 dof Full Order Observer)

3a) Copy your program **DT_sv1_driver.m** to a new file **DT_sv1_observer_driver.m**, then copy the Simulink file **DT_sv1.mdl** to **DT_sv1_observer.mdl**. Modify these *new* files to implement state variable feedback using a full order observer to estimate the states for a one degree of freedom system. Your programs should be able to use either **place** or **acker** to place the closed loop poles directly, or **dlqr** to place the poles using the LQR algorithm. Your programs should write all of the true states and all of the estimated states to the workspace. Note that if our system has no integrator in it, there should be a prefilter. Be sure $u(k)$ is taken to be the signal just before the limiter.

3b) Modify **DT_sv1_observer_driver.m** to plot the first two true states (not the delayed control signal) and estimated states on the same graph, as well as the system output and the control effort.

3c) Place both the state feedback and observer poles all at 0.5 (yes, this is a bad design) The output of the system is the position of the first disk and this is also the output available to the observer ($C = C_y$ in this case). The disk is initially estimated to be at 5 degrees (be careful of units here! The model works in radians.). The initial estimates of the velocity and delayed control signal are zero. The real system starts at rest (all initial conditions are zero.) For a 10 degree step input you should get the graph shown in Figure 1. Turn in your plot.

3d) Now place the observer poles all at 0.3, then at 0.2, then at 0. Rerun the system and turn in your plots. You should see the estimated states converging mores quickly to the real states as the observer poles get closer to the origin.

3e) Now assume the same condition as in part c, but place the observer poles at 0.45, 0.5, and 0.55 (you will need to use the **place** command to place the poles.) Assume both the position of the first disk (state 1) and the delayed control signal (state 3) are available to the observer. (*Hint: C is a 2x3 matrix*) However, the output is still the position of the first disk. Simulate your system, you should get a result like that in Figure 2. Note that the observer has more information to work with here, so it does a better job of estimating the states. Turn in your plot.

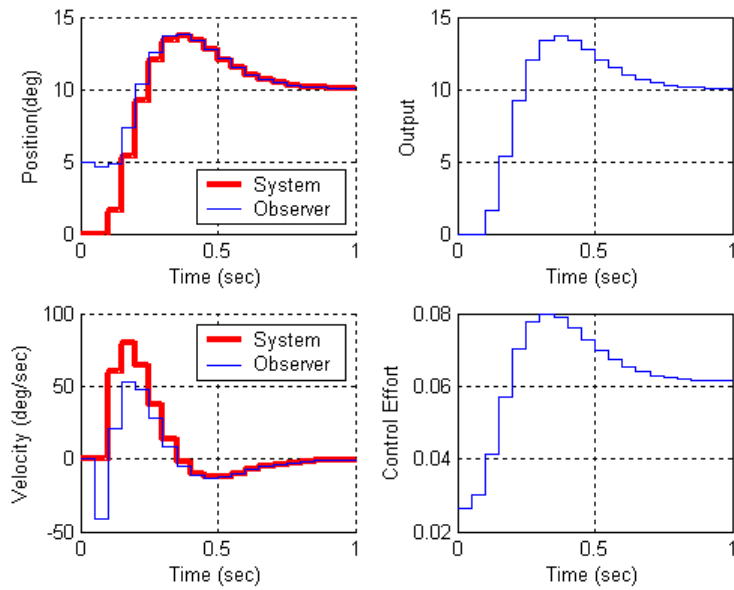


Figure 1. Results for problem 3c.

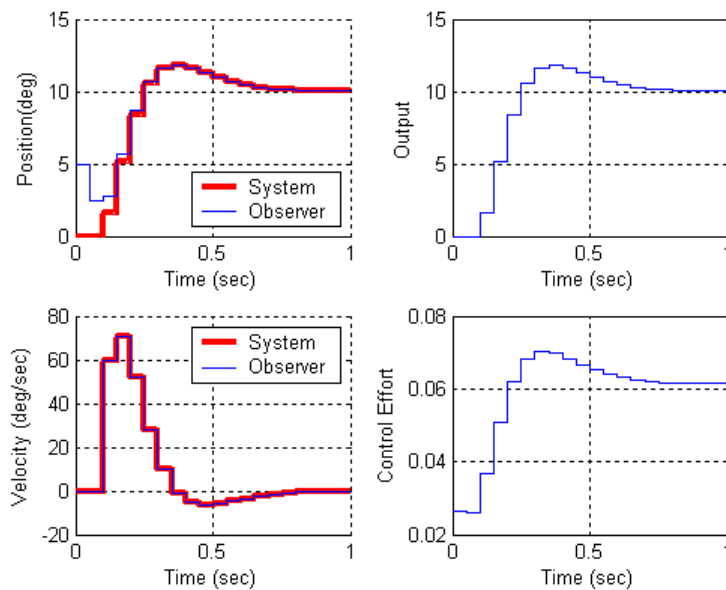


Figure 2. Results for part 3e.

(2 dof Full Order Observer)

4a) Copy your program `DT_sv1_observer_driver.m` to a new file `DT_sv2_observer_driver.m`, then copy the Simulink file `DT_sv1_observer.mdl` to `DT_sv2_observer.mdl`. Modify these *new* files to implement both state variable feedback using a full order observer to estimate the states for a two degree of freedom system. Your programs should be able to use either `place` or `acker` to place the closed loop poles directly, or

dlqr to place the poles using the LQR algorithm. Your programs should write all of the true states and all of the estimated states to the workspace. Note that if our system has no integrator in it, there should be a prefilter. Be sure $u(k)$ is taken to be the signal just before the limiter.

4b) Modify **DT_sv2_observer_driver.m** to plot the first four true states (not the delayed control signal) and estimated states on the same graph, as well as the system output and the control effort.

4c) Place all the state feedback poles at 0.2, and the observer poles at 0.06 0.08 0.1 0.12 0.14. You will probably need to use the **place** command to place the observer poles here. Assume the positions of both the first and second disk are available to the observer and the system output is the position of the first disk. The velocities of the disks and the delayed control signals are initially estimated to be zero. The estimated position of the first disk is assumed to be -5 degrees and the estimated position of the second disk is assumed to be 5 degrees. The real system is starts at rest. For a 10 degree step input you should get the graph shown in Figure 3. Turn in your plot.

4d) For the same conditions as part d, but assume the observer can only use the position of the second disk. You should get the results shown in Figure 4. Turn in your plot.

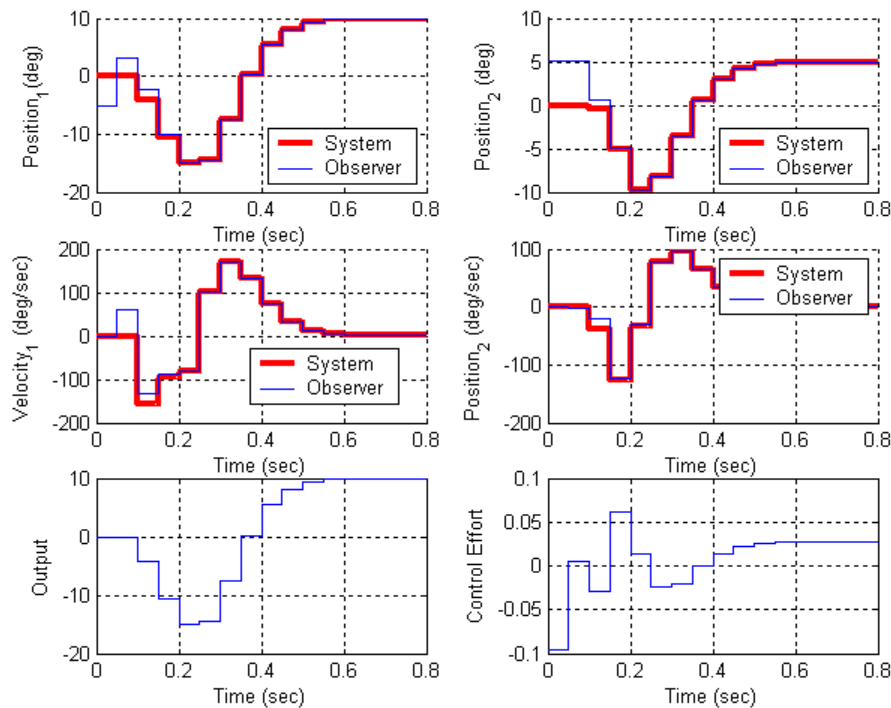


Figure 3: Results for problem 4c.

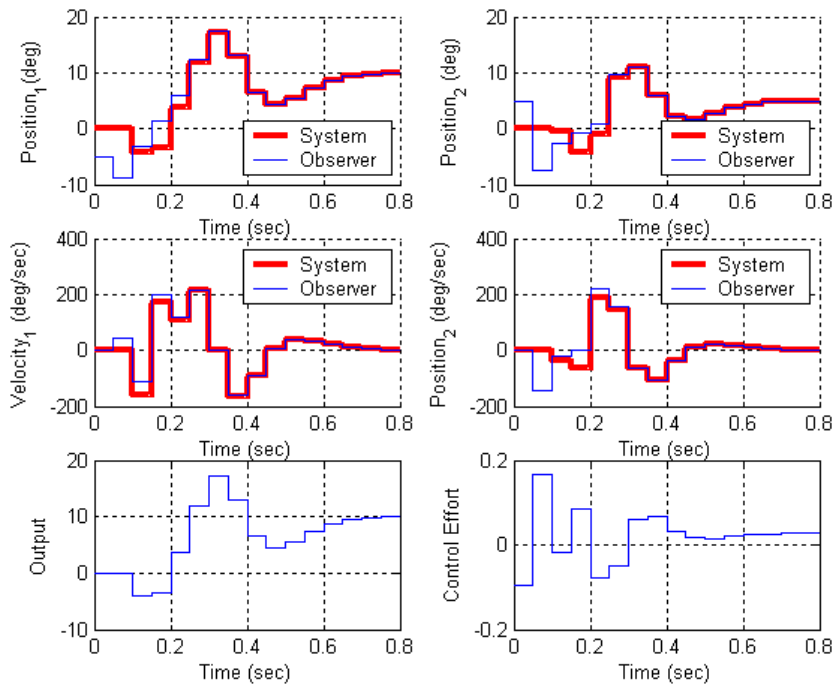


Figure 4: Results for problem 4d.

(1 dof Integral Control and Full Order Observer)

5) Now we want to combine the observer and state feedback with integral control for the one degree of freedom system. We will continue to utilize the torsional model we have been using.

5a) Copy your program **DT_sv1_observer_driver.m** to a new file **DT_sv1_obs_servo_driver.m**, then copy the Simulink file **DT_sv1_observer.mdl** to **DT_sv1_obs_servo.mdl**. Modify these new files to implement both state variable feedback using a full order observer to estimate the states and integral control.

5b) Place all the state feedback poles at 0.4 and all of the observer poles at 0.2. The output of the system is the position of the first disk and this is also the output available to the observer ($C = C_y$ in this case). The disk is initially estimated to be at 5 degrees, the velocity is initially estimated to be -5 degrees/sec, and the delayed control signal is estimated to be 1 degree. The true system is initially at rest (all initial states are zero) For a 10 degree step input you should get the graph shown in Figure 5. Turn in your plot.

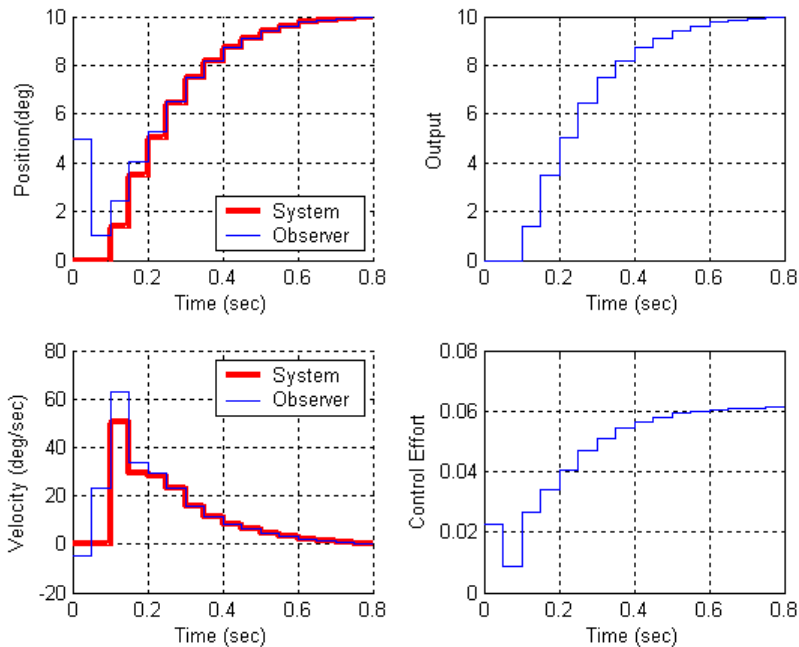


Figure 5: Results for problem **5b**.

(2dof Integral Control and Full Order Observer)

6) Finally, we want to combine the observer and state feedback with integral control for the two degree of freedom system. We will continue to utilize the torsional model we have been using.

6a) Copy your program **DT_sv2_observer_driver.m** to a new file **DT_sv2_obs_servo_driver.m**, then copy the Simulink file **DT_sv2_observer.mdl** to **DT_sv2_obs_servo.mdl**. Modify these new files to implement both state variable feedback using a full order observer to estimate the states and integral control. Note that if our system has no integrator in it, there should be a prefilter. Be sure $u(k)$ is taken to be the signal just before the limiter.

6b) Place all the state feedback poles at 0.2, and the observer poles at 0.06 0.08 0.1 0.12 0.14. You will probably need to use the **place** command to place the observer poles here. Assume the position of the second disk is available to the observer and the system output is the position of the second disk. The estimated position of the first disk is assumed to be 5 degrees and the estimated position of the second disk is assumed to be -5 degrees. All other initial estimates are zero. The real system is starts at rest. The velocity and control inputs are assume to be zero. For a 10 degree step input you should get the graph shown in Figure 6. Turn in your plot.

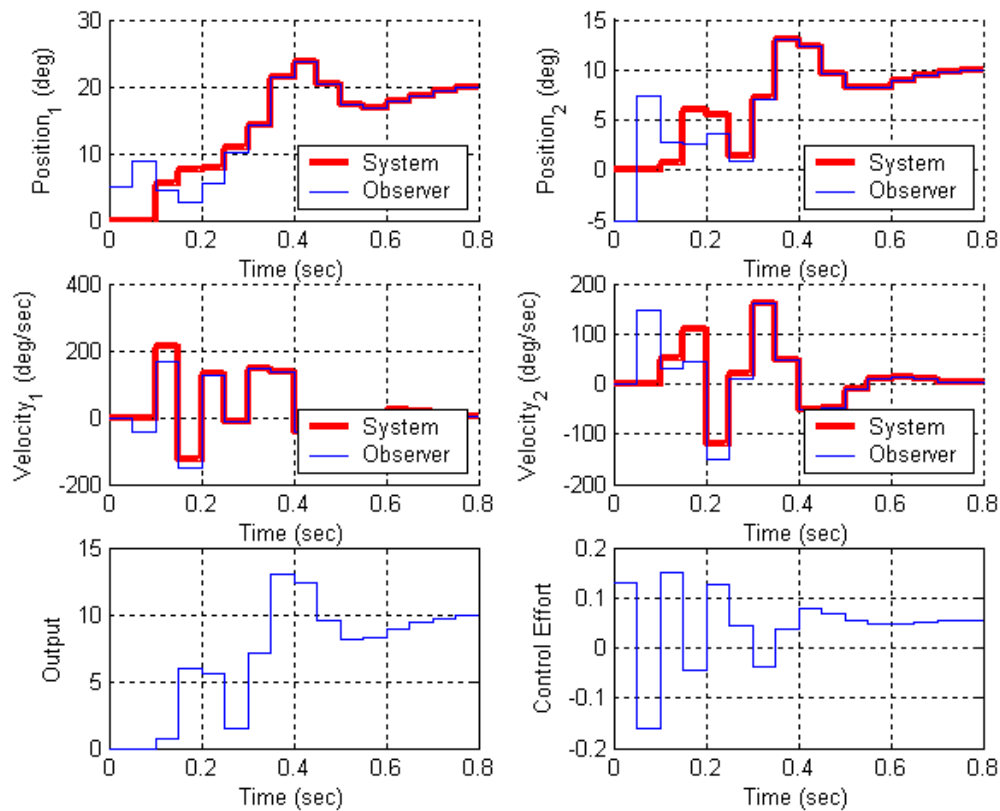
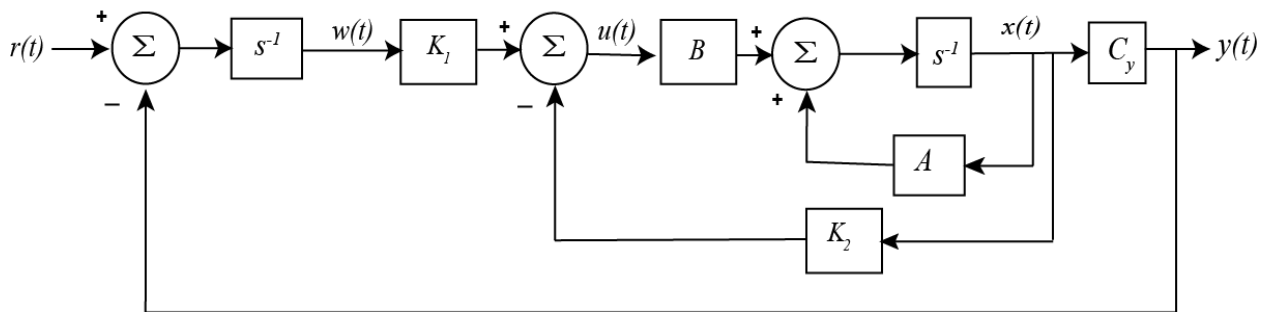


Figure 6: Results for problem 6b.

7) In class we showed how to make a discrete-time state variable feedback system into a type one system. In this problem we do the same thing for a continuous-time system.

Consider the following continuous-time state variable system:



For this system we have

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= C_y x(t) \\ u(t) &= -K_2 x(t) + K_1 w(t) \\ \dot{w}(t) &= r(t) - y(t) \end{aligned}$$

- a) Show that we can represent the open loop system in the following form, and determine the α_i

$$\begin{bmatrix} \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} \alpha_5 \\ \alpha_6 \end{bmatrix} u(t) + \begin{bmatrix} \alpha_7 \\ \alpha_8 \end{bmatrix} r(t)$$

- b) Define

$$x_e(t) = x(t) - x(\infty)$$

$$w_e(t) = w(t) - w(\infty)$$

$$u_e(t) = u(t) - u(\infty)$$

and assume $r(t) = r(\infty) = 1$. Show that we can write this new system in the following form and determine the β_i

$$\begin{bmatrix} \dot{x}_e(t) \\ \dot{w}_e(t) \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{bmatrix} \begin{bmatrix} x_e(t) \\ w_e(t) \end{bmatrix} + \begin{bmatrix} \beta_5 \\ \beta_6 \end{bmatrix} u_e(t)$$

- c) Show that we can write $u_e(t) = -K_2 x_e(t) + K_1 w_e(t)$. Don't just use the results from part a, you also need to use part b.

- d) Now define

$$e(t) = \begin{bmatrix} x_e(t) \\ w_e(t) \end{bmatrix}$$

Show that we can now write the equations

$$\dot{e}(t) = \hat{A}e(t) + \hat{B}u_e(t)$$

$$u_e(t) = -\hat{K}e(t)$$

What are the \hat{A} , \hat{B} , and \hat{K} ?

- e) Show that we can then write $\dot{e}(t) = (\hat{A} - \hat{B}\hat{K})e(t)$ which is the standard form for pole placement of a state variable feedback system.

- f) For the state variable model

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -175 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 5050 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0]x(t)$$

implement a type one state variable feedback system in Simulink with a Matlab driver file. If the pole are placed at -10, -20, -30 with the Matlab place command, you should get results like

that in Figure 7. Turn in your graph and e-mail me both the Simulink file and the Matlab driver file.

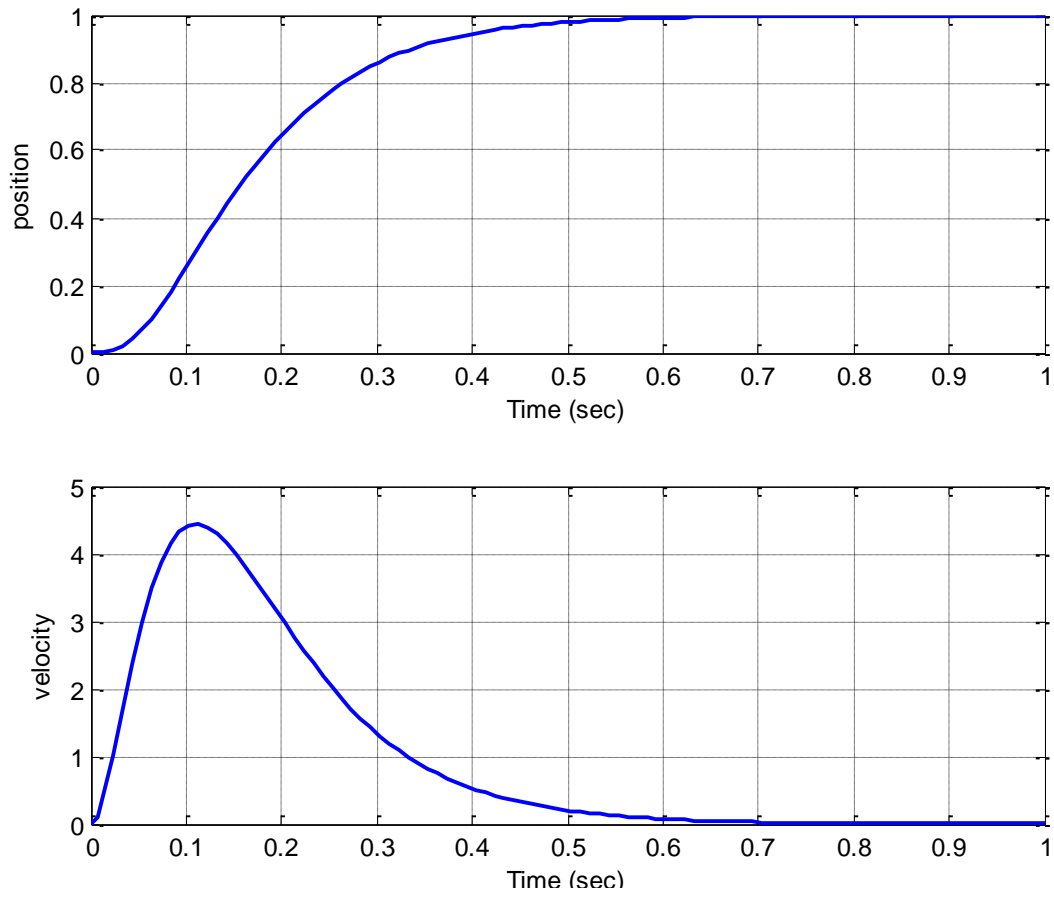


Figure 7. Results for a continuous time state variable system with an integrator