ECE-420: Discrete-Time Control Systems
Homework 1

Due: Friday September 12 at the beginning of class

1) From class we have the useful sum $S_N = \sum_{k=0}^{k=N} a^k = \frac{1-a^{N+1}}{1-a}$. Using this sum, and possibly a change of variables, show that

a) $\sum_{k=0}^{k=N} \left(\frac{a}{b}\right)^k = \frac{b^{N+1} - a^{N+1}}{b^N (b-a)}$

b) $\sum_{k=M}^{k=N} a^k = \frac{a^M - a^{N+1}}{1-a}$

2) Starting from $S_N = \sum_{k=0}^{k=N} a^k = \frac{1-a^{N+1}}{1-a}$, take derivatives of both sides to show that

$$\sum_{k=0}^{k=N} k a^k = \frac{N a^{N+2} - (N+1) a^{N+1} + a}{(1-a)^2}$$

3) For impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ and input $x(n) = u(n)$, show that the system output is $y(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$ by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} x(n-k) h(k)$

Note that this is the unit step response of the system.

4) For impulse response $h(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3)$ and input $x(n) = \left(\frac{1}{2}\right)^{n-1} u(n-2)$, determine the output $y(n)$ (this should be easy).

5) Show that $u(n) = \sum_{l=-\infty}^{l=n} \delta(l)$ and $u(n-k) = \sum_{l=-\infty}^{l=n} \delta(l-k)$

6) For impulse response $h(n)=\left(\frac{1}{3}\right)^{n-2}u(n-1)$ and input $x(n)=\left(\frac{1}{2}\right)^{n}u(n-1)$, show that the system output is $y(n)=9\left[\left(\frac{1}{2}\right)^{n-1}-\left(\frac{1}{3}\right)^{n-1}\right]u(n-2)$ by evaluating the convolution sum

$$y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$$

7) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n-3}u(n-1)$ and input $x(n)=\left(\frac{1}{4}\right)^{n+1}u(n-2)$, show that the system output is $y(n)=\left[\left(\frac{1}{2}\right)^{n}-\left(\frac{1}{4}\right)^{n-1}\right]u(n-3)$ by evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}x(n-k)h(k)$

8) For impulse response $h(n)=\left(\frac{1}{5}\right)^{n}u(n)$ and input $x(n)=u(-n)$, show that the system output is $y(n)=\frac{5}{4}u(-n)+\frac{1}{4}\left(\frac{1}{5}\right)^{n-1}u(n-1)$ by evaluating the convolution sum $y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$

9) For impulse response $h(n)=\left(\frac{1}{3}\right)^{n-2}u(n-3)$ and input $x(n)=\left(\frac{1}{2}\right)^{-n}u(2-n)$, show that the system output is $y(n)=\frac{1}{5}2^{n-2}u(5-n)+\frac{8}{5}\left(\frac{1}{3}\right)^{n-5}u(n-6)$ by evaluating the convolution sum

$$y(n)=\sum_{k=-\infty}^{\infty}h(n-k)x(k)$$