Name Solutions Mail Box

ECE-420
Exam 2

Fall 2014

No Calculators allowed. You must show your work to receive credit.

Problem 1	**______/30**
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Problem 3	**______/10**
Problem 4	**______/30**

Total ______

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1) Assume we have the following state variable system

$$x(n+1)=\begin{bmatrix}0 & 1\\ 1 & 2\end{bmatrix}x(n)+\begin{bmatrix}1\\ 0\end{bmatrix}u(n)$$

$$y(n)=\begin{bmatrix}1 & 0\end{bmatrix}x(n)$$

With state variable feedback

$$u(n)=G_{pf}r(n)-Kx(n)$$

where $r(n)$ is the reference input, G_{pf} is the prefilter gain, and K is the state variable feedback vector.

a) Determine the transfer function between $R(z)$ and $Y(z)$ in terms of k_1, k_2, and G_{pf}

b) Using your result from part (a) determine k_1 and k_2 so the closed loop poles are at -1 and -2.

c) Using your results from (a) and (b) determine the prefilter gain G_{pf} so the steady state error for a unit step is zero.

d) Use the direct eigenvalue assignment method ($[\lambda I - G \mid H]$) to determine k_1 and k_2 so the closed loop poles are at -1 and -2. Do you get the same answer as for part (b)?

a) $\tilde{G} = G - HK = \begin{bmatrix}0 & 1\\ 1 & 2\end{bmatrix} - \begin{bmatrix}1\\ 0\end{bmatrix}\begin{bmatrix}K_1 & K_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 2\end{bmatrix} - \begin{bmatrix}K_1 & K_2\\ 0 & 0\end{bmatrix} = \begin{bmatrix}-K_1 & 1-K_2\\ 1 & 2\end{bmatrix}$

$zI - \tilde{G} = \begin{bmatrix}z & 0\\ 0 & z\end{bmatrix} - \begin{bmatrix}-K_1 & 1-K_2\\ 1 & 2\end{bmatrix} = \begin{bmatrix}z+K_1 & K_2-1\\ -1 & z-2\end{bmatrix}$ $\quad (zI-\tilde{G})^{-1} = \frac{1}{\Delta(z)}\begin{bmatrix}z-2 & 1-K_2\\ 1 & z+K_1\end{bmatrix}$

$\Delta(z) = (z+K_1)(z-2) + K_2 - 1 = \boxed{z^2 + (K_1-2)z - 2K_1 + K_2 - 1 = \Delta(z)}$

$G_0(z) = C(zI-\tilde{G})^{-1}\tilde{H} = \begin{bmatrix}1 & 0\end{bmatrix}\frac{1}{\Delta(z)}\begin{bmatrix}z-2 & 1-K_2\\ 1 & z+K_1\end{bmatrix}\begin{bmatrix}G_{pf}\\ 0\end{bmatrix} = \boxed{\frac{(z-2)G_{pf}}{\Delta(z)} = G_0(z)}$

b) desired $\Delta(z) = (z+1)(z+2) = z^2 + 3z + 2 = z^2 + (K_1-2)z + K_2 - 2K_1 - 1$

$K_1 - 2 = 3 \qquad K_2 - 2K_1 - 1 = 2$

$\boxed{K_1 = 5} \qquad K_2 = 2 + 2K_1 + 1 = \boxed{13 = K_2}$

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#1 c) $G_0(1) = 1 = \frac{(1-2)G_{pf}}{1+3+2} = -\frac{G_{pf}}{6} = 1$ $\boxed{G_{pf} = -6}$

d) $\lambda_1 = -1$ $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & -3 \end{bmatrix}$

$\begin{bmatrix} -1 & -1 & 1 \\ -1 & -3 & 0 \end{bmatrix} \underline{\xi}^1 = 0$ $\underline{\xi}^1 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

$\lambda_2 = -2$ $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -1 & -4 \end{bmatrix}$

$\begin{bmatrix} -2 & -1 & 1 \\ -1 & -4 & 0 \end{bmatrix} \underline{\xi}^2 = 0$ $\underline{\xi}^2 = \begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}$

$K \begin{bmatrix} 3 \\ -1 \end{bmatrix} = 2$ $K \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 7$

$K \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 7 \end{bmatrix}$ $K = \begin{bmatrix} 2 & 7 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}^{-1}$ $\begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$

$K = \begin{bmatrix} 2 & 7 \end{bmatrix} \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 13 \end{bmatrix}$

$\boxed{K_1 = 5 \quad K_2 = 13}$

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2) We have been assuming the following form for our discrete-time transfer functions with input $R(z)$ and output $Y(z)$

$$G_p(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Cross multiplying we get

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) = b_0 U(z) + b_1 z^{-1} U(z) + b_2 z^{-2} U(z)$$

In the time-domain this becomes

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_0 u(n) + b_1 u(n-1) + b_2 u(n-2)$$

Assume we have the following time-varying plant

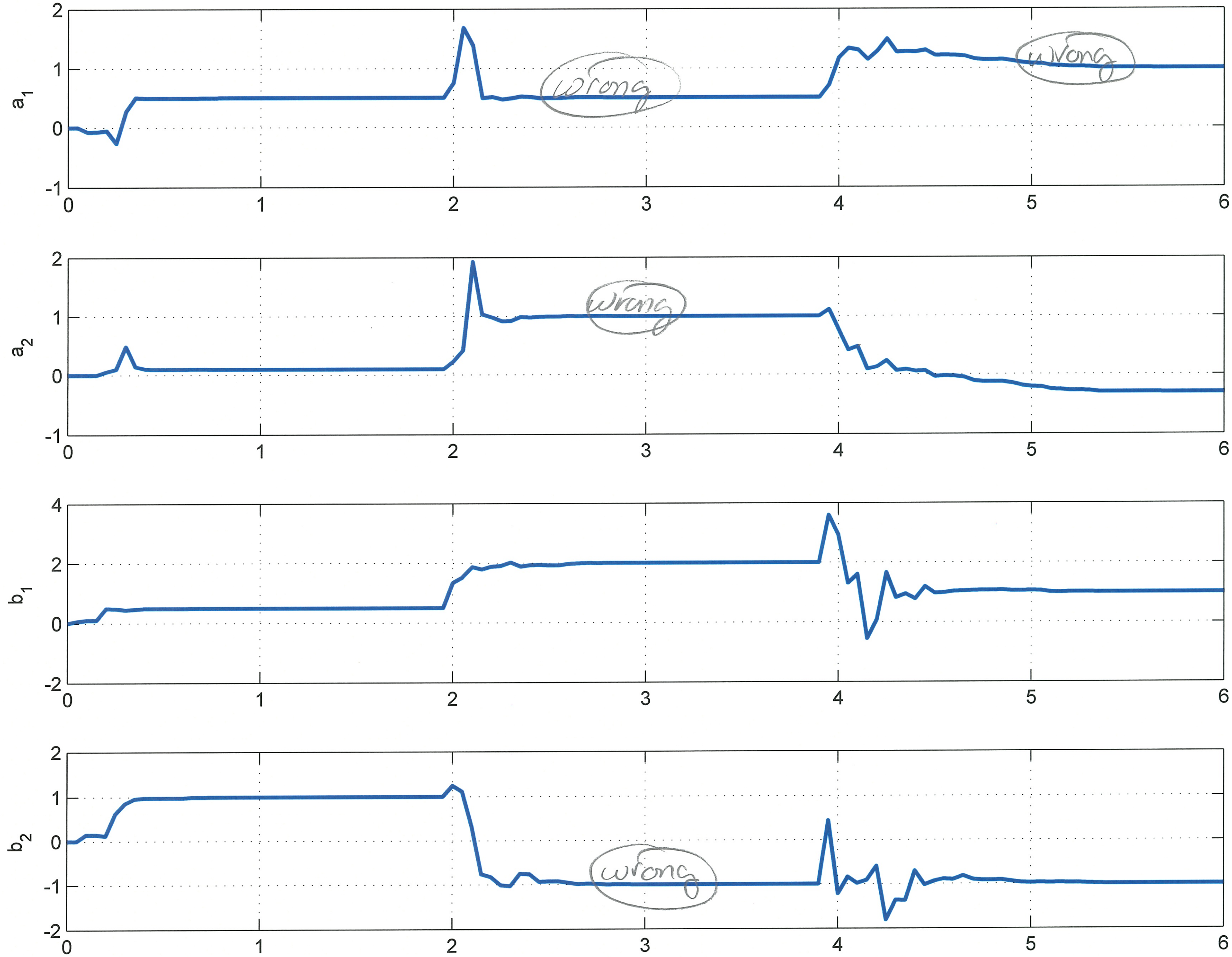
$$G_p(z) = \frac{0.5z^{-1} + 1z^{-2}}{1 + 0.5z^{-1} + 0.1z^{-2}} \qquad t < 2$$

$$G_p(z) = \frac{2z^{-1} + 1z^{-2}}{1 - 0.2z^{-1} + 0.3z^{-2}} \qquad 2 < t < 4$$

$$G_p(z) = \frac{1z^{-1} - 1z^{-2}}{1 + 0z^{-1} - 0.3z^{-2}} \qquad t > 4$$

We utilize a recursive least squares algorithm and get the results shown in the graph on the next page. Is the algorithm working or not? Explain you answer. If the algorithm is failing indicate all the ways you feel the algorithm is failing. If the algorithm is working, explain how the performance might be improved.

It is not working, it fails to converge to 4 values (see next page) λ = 0.7



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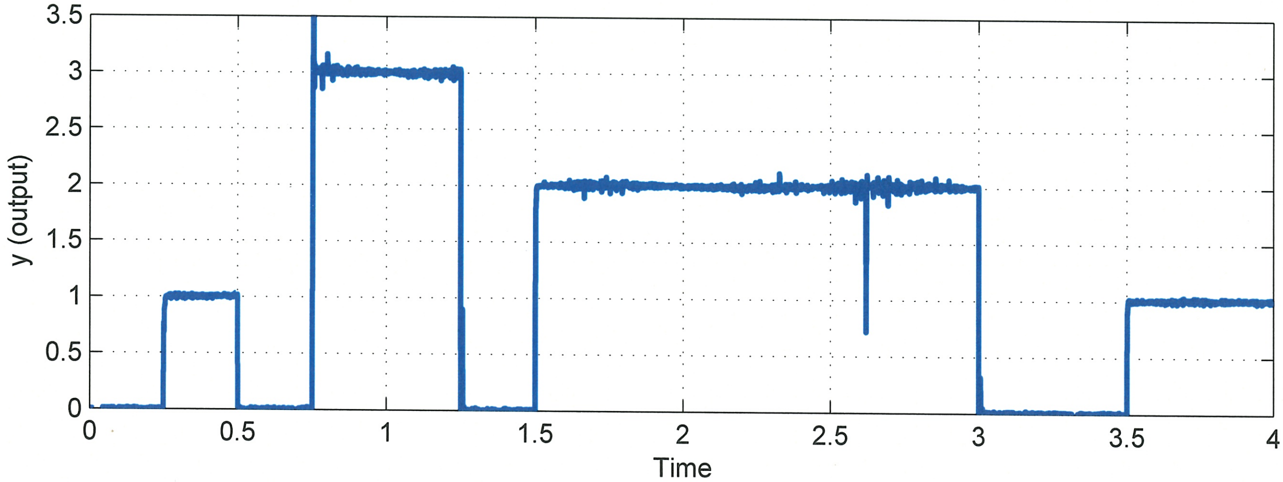
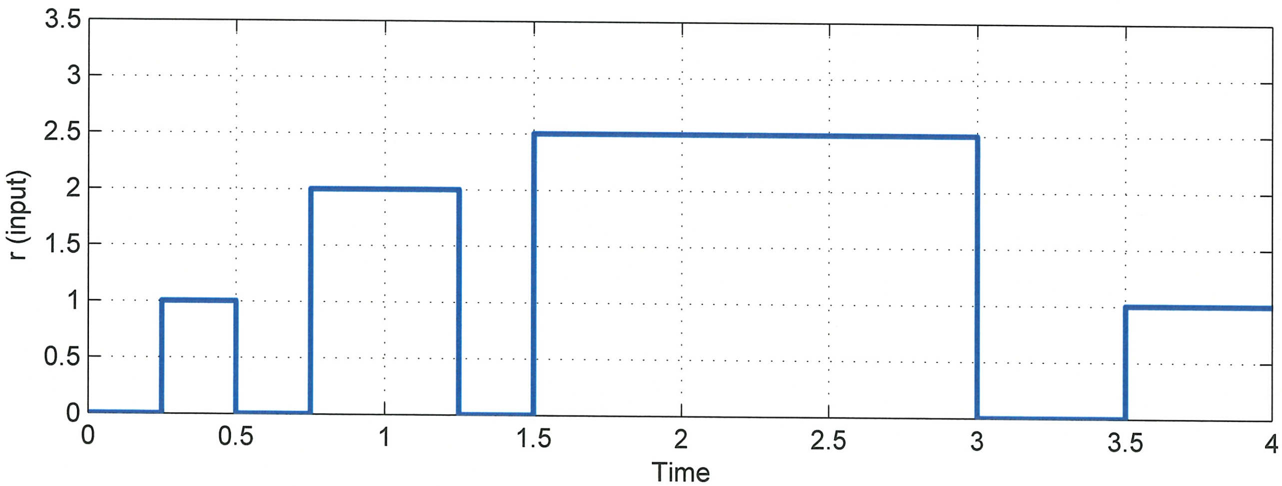
3) The input and output from an adaptive controller are shown on the following page. Is this adaptive controller working in general? Why or why not? (do not worry/include the occasional spikes in your answer)

For a control system the output should match the input, as much as possible. The output does not match the input here so the answer is no


3.5
3
2.5
2
1.5
1
0.5
0
y (output)
0
0.5
1
1.5
2
2.5
3
3.5
4
Time


3.5
3
2.5
2
1.5
1
0.5
0
r (input)
0
0.5
1
1.5
2
2.5
3
3.5
4
Time

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4) For this problem, assume $\underline{p} = \begin{bmatrix} a \\ b \end{bmatrix}, \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ and we are **given** the following (you can just use these):

i) for $f(\underline{x}) = \underline{p}^T \underline{x}, \dfrac{df}{d\underline{x}} = \underline{p}^T$

ii) for $f(\underline{x}) = \underline{x}^T \underline{p}, \dfrac{df}{d\underline{x}} = \underline{p}^T$

iii) for $f(\underline{x}) = A\underline{x}, \dfrac{df}{d\underline{x}} = A$

iv) for $f(\underline{x}) = A^T \underline{x}, \dfrac{df}{d\underline{x}} = A^T$

v) for $f(\underline{x}) = \underline{x}^T A\underline{x}, \dfrac{df}{d\underline{x}} = x^T (A + A^T)$

Consider the overdetermined system $Ax = b$.Rather than just performing a normal least squares solution, we want to penalize the magnitude of the solution by some scalar μ . Thus we want to find the value of x that minimizes

$$H = \| Ax - b \|^2 + \mu \| x \|^2 = (Ax - b)^T (Ax - b) + \mu \, (x^T Ix)$$

a) Show that the solution to the above minimization problem is given by $x = (A^T A + \mu I)^{-1} A^T b$

b) Assume now we compute the singular value decomposition of A , $A = USV^T$. Recall that U and V are unitary matrices, so $U^T U = V^T V = I$. This means that $U^{-1} = U^T$ and $V^{-1} = V^T$. S is a diagonal matrix with elements σ_i on the diagonal (assume S has full rank and is invertible). Assume that we can write $b = U\beta$ and $x = V\alpha$. Determine how α and β are related. *Hint: it may be helpful to write* $I = VIV^T$

Cultural Point: This technique is known as zero order Tikhonov regularization.

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#4 a) $H = (Ax-b)^T(Ax-b) + u\, x^T I x$

$= (x^T A^T - b^T)(Ax-b) + u x^T I x$

$= x^T A^T A x - b^T A x - x^T A^T b + b^T b + u x^T I x$

$\frac{dH}{dx} = 2x^T A^T A - b^T A - b^T A + 2u x^T I = 0$

$2x^T A^T A - 2b^T A + 2u x^T I = 0$

$(A^T A + uI)x = A^T b$

$\boxed{x = (A^T A + uI)^{-1} A^T b}$

b) $(V\alpha) = \left[(USV^T)^T(USV^T) + u(VIV^T)\right]^{-1}\left[USV^T\right]^T U\beta$

$= \left[VS^T U^T U S V^T + uVIV^T\right]^{-1} VS^T U^T U\beta$

$= \left[VS^T S V^T + VuIV^T\right]^{-1} VS^T\beta$

$= \left[V(S^T S + uI)V^T\right]^{-1} VS^T\beta$

$= \left[V(S^T S + uI)^{-1} V^T\right] VS^T\beta$

$= V(S^T S + uI)^{-1} S^T\beta \quad = V\alpha$

$\boxed{\alpha = (S^T S + uI)^{-1} S^T\beta}$