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ECE-420
Exam 1

Fall 2014

Calculators and Laptops cannot be used. You must show your work to receive credit.

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Total _______________

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1a) Use long division to determine the first three nonzero terms in the impulse response for the following transfer function $H(z) = \frac{z+2}{z^2-1}$

$$
\begin{array}{r|l}
 & z^{-1} + 2z^{-2} + z^{-3} \\
z^2 - 1 & z + 2 \\
 & z \quad - z^{-1} \\
\hline
 & 2 + z^{-1} \\
 & 2 \quad - 2z^{-2} \\
\hline
 & z^{-1} + 2z^{-2}
\end{array}
$$

$$\boxed{h(n) = \delta(n-1) + 2\,\delta(n-2) + 3\delta(n-3) + \cdots}$$

1b) Assume a system has impulse response $h(n) = (0.5)^n u(n)$ Determine the output if the input is $x(n) = \delta(n) + 3\delta(n-1)$

$$y(n) = \sum_{K=-\infty}^{\infty} h(n-K)\,x(K) = \sum_{K=-\infty}^{\infty} (0.5)^{n-K} u(n-K)\left[\delta(K) + 3\,\delta(K-1)\right]$$

$$= \sum_{K=-\infty}^{\infty} (0.5)^{n-K} u(n-K)\,\delta(K) + 3\sum_{K=-\infty}^{\infty} (0.5)^{n-K} u(n-K)\,\delta(K-1)$$

$$= \boxed{(0.5)^{n} u(n) + 3(0.5)^{n-1} u(n-1) = y(n)}$$

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2) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n}u(n-2)$ and input $x(n)=\left(\frac{1}{3}\right)^{n-2}u(n)$

a) Determine $H(z)$

b) Determine $X(z)$

c) Assume $Y(z)=z^{-1}G(z)$, determine $g(n)$ and then $y(n)$

$$h(n)=\left(\tfrac{1}{2}\right)^{n-2}\left(\tfrac{1}{2}\right)^{2}u(n-2) \qquad H(z)=z^{-2}\left(\tfrac{1}{2}\right)^{2}\frac{z}{z-\frac{1}{2}}=\boxed{\frac{\frac{1}{4}z^{-1}}{z-\frac{1}{2}}=H(z)}$$

$$x(n)=\left(\tfrac{1}{3}\right)^{n}\left(\tfrac{1}{3}\right)^{-2}u(n) \qquad X(z)=\left(\tfrac{1}{3}\right)^{-2}\frac{z}{z-\frac{1}{3}}=\boxed{\frac{9z}{z-\frac{1}{3}}=X(z)}$$

$$Y(z)=H(z)X(z)=\frac{9/4}{(z-\frac{1}{2})(z-\frac{1}{3})} \qquad G(z)=\frac{\frac{9}{4}z}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$\frac{G(z)}{z}=\frac{9/4}{(z-\frac{1}{2})(z-\frac{1}{3})}=\frac{A}{z-\frac{1}{2}}+\frac{B}{z-\frac{1}{3}} \qquad A=\frac{9/4}{1/6}=\frac{27}{2}$$

$$B=\frac{9/4}{-1/6}=-\frac{27}{2}$$

$$g(n)=\frac{27}{2}\left[\left(\tfrac{1}{2}\right)^{n}-\left(\tfrac{1}{3}\right)^{n}\right]u(n)$$

$$y(n)=g(n-1)=\boxed{\frac{27}{2}\left[\left(\tfrac{1}{2}\right)^{n-1}-\left(\tfrac{1}{3}\right)^{n-1}\right]u(n-1)=y(n)}$$

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3) Consider the continuous-time plant $G_p(s) = \frac{A}{s}$. Determine the equivalent discrete-time plant $G_p(z)$ assuming a zero-order hold (ZOH) is placed before the continuous-time plant to convert the discrete-time control signal to a continuous-time control signal.

$$G_i(z) = \mathcal{Z}\left\{\frac{G_p(s)}{s}\right\} = \mathcal{Z}\left\{\frac{A}{s^2}\right\} = \frac{ATz}{(z-1)^2}$$

$$G_p(z) = (1 - z^{-1})\, G_i(z) = \frac{z-1}{z} \cdot \frac{ATz}{(z-1)^2} = \boxed{\frac{AT}{z-1} = G_p(z)}$$

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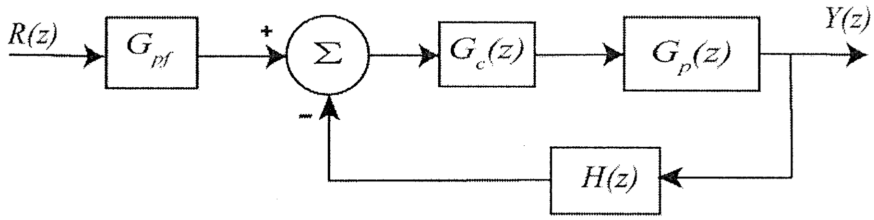
4) Assume the following feedback configuration



If $H(z) = z^{-1}, G_c(z) = \dfrac{c(z+a)}{(z-1)(z+b)}, G_p(z) = \dfrac{2z}{z+1}$ determine the parameters a, b, and c if the desired closed loop poles are roots of the equation $\Delta(z) = z^3 + d_1 z^2 + d_2 z + d_3$

$$\Delta(z) = 1 + H(z)\, G_p(z)\, G_c(z) = 1 + z^{-1} \frac{2z}{z+1} \frac{c(z+a)}{(z-1)(z+b)} = 0$$

$$\Delta(z) = (z+1)(z-1)(z+b) + 2c(z+a)$$

$$= (z^2-1)(z+b) + 2cz + 2ca = z^3 + bz^2 - z - b + 2cz + 2ca$$

$$= z^3 + bz^2 + (2c-1)z + (2ca-b) = z^3 + d_1 z^2 + d_2 z + d_3$$

$$\boxed{b = d_1}$$

$$2c - 1 = d_2 \qquad 2c = d_2 + 1 \qquad \boxed{c = \frac{d_2+1}{2}}$$

$$2ca - b = d_3 \qquad 2ca = d_3 + b = d_3 + d_1$$

$$a = \frac{d_3 + d_1}{2c} = \boxed{\frac{d_3 + d_1}{d_2 + 1} = a}$$

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5) For impulse response $h(n)=\left(\frac{1}{2}\right)^{n}u(n-2)$ and input $x(n)=\left(\frac{1}{2}\right)^{-n}u(-\text{n})$, the system output can be written as $A(n)u(n-3)+B(n)u(2-n)$. Determine an expression for **both** $A(n)$ **and** $B(n)$. You do not need to simplify your expressions but you must evaluate all sums.

$$y(n)=\sum_{K=-\infty}^{\infty} h(n-K)x(K)=\sum_{K=-\infty}^{\infty}\left(\tfrac{1}{2}\right)^{n-K}u(n-K-2)\left(\tfrac{1}{2}\right)^{-K}u(-K)$$

$u(n-K-2)=1$ for $n-K-2\geq 0$ or $\boxed{n-2\geq K}$ the same for

$u(-K)=1$ for $-K\geq 0$ or $\boxed{0\geq K}$ $n=2$

$\underline{n\geq 3}$ $\quad A(n)=\sum_{K=-\infty}^{0}\left(\tfrac{1}{2}\right)^{n}\left(\tfrac{1}{2}\right)^{-2K}=\left(\tfrac{1}{2}\right)^{n}\sum_{K=-\infty}^{0}\left(\tfrac{1}{4}\right)^{-K}$

let $l=-K$ $\quad A(n)=\left(\tfrac{1}{2}\right)^{n}\sum_{l=0}^{\infty}\left(\tfrac{1}{4}\right)^{l}=\left(\tfrac{1}{2}\right)^{n}\frac{1}{1-1/4}=\boxed{\frac{4}{3}\left(\tfrac{1}{2}\right)^{n}=A(n)}$

$\underline{n\leq 2}$ $\quad B(n)=\sum_{K=-\infty}^{n-2}\left(\tfrac{1}{2}\right)^{n}\left(\tfrac{1}{4}\right)^{-K}$ $\quad$ let $l=n-2-K$ $\quad -K=l-n+2$

$$=\left(\tfrac{1}{2}\right)^{n}\sum_{l=\infty}^{0}\left(\tfrac{1}{4}\right)^{l-n+2}=\left(\tfrac{1}{2}\right)^{n}\left(\tfrac{1}{4}\right)^{-n}\left(\tfrac{1}{4}\right)^{2}\sum_{l=0}^{\infty}\left(\tfrac{1}{4}\right)^{l}$$

$$=2^{n}\left(\tfrac{1}{4}\right)^{2}\frac{1}{1-1/4}=\frac{4}{3}2^{n-4}=\boxed{\frac{1}{3}2^{n-2}=B(n)}$$