Name____ Mail Box __________________

ECE-420
Exam 2

Fall 2013

No Calculators allowed. You must show your work to receive credit.

Problem 1 __________/15

Problem 2 __________/30

Problem 3 __________/30

Problem 4 __________/25

Total ________________

Name____ Mail Box __________________

1) Assume we have a process which we expect to be able to model as $g(t) = \alpha t e^{-\beta t}$. We perform an experiment and determine values for $g(t)$ for a number of different values of t. *Assume none of the values of g(t) or t are zero*. So we have the values

$$
\begin{array}{ll}
t_0, & g(t_0) \\
t_1, & g(t_1) \\
t_2, & g(t_2) \\
t_3, & g(t_3) \\
t_4, & g(t_4)
\end{array}
$$

We want to estimate α and β using a least squares type of approach, with the model $z = Aw$. Describe how you would do this. Be sure in your description, to indicate what the z, w, and A are in terms of the values given $t_i, g(t_i)$

If you cannot solve for α and β directly, tell me how you would get them.

Name____ Mail Box ___________________

2) Consider the discrete-time state variable model

$$\underline{x}(k+1) = G\underline{x}(k) + Hu(k)$$

with the initial state $x(0) = 0$. Let

$$G = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0$$

a) Determine the characteristic equation for G

b) Show that we can write $x(4) = \begin{bmatrix} GH & H \end{bmatrix} \tilde{u}(3)$ and determine an expression for $\tilde{u}(3)$

c) Is this system controllable?

Name____ Mail Box ___________________

3) For this problem, assume $\underline{p} = \begin{bmatrix} a \\ b \end{bmatrix}, \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ and we are **<u>given</u>** the following (you can just use these):

i) for $f(\underline{x}) = \underline{p}^T \underline{x}, \dfrac{df}{d\underline{x}} = \underline{p}^T$

ii) for $f(\underline{x}) = \underline{x}^T \underline{p}, \dfrac{df}{d\underline{x}} = \underline{p^T}$

iii) for $f(\underline{x}) = A\underline{x}, \dfrac{df}{d\underline{x}} = A$

iv) for $f(\underline{x}) = A^T \underline{x}, \dfrac{df}{d\underline{x}} = A^T$

v) for $f(\underline{x}) = \underline{x}^T A\underline{x}, \dfrac{df}{d\underline{x}} = x^T (A + A^T)$

Consider the underdetermined system $Ax = b$. Note that there are more unknowns than equations. We want to determine the value of x that satisfies this equation and minimizes $x^T Rx$. You may assume that R is symmetric and has an inverse.

Hint: You will need to use a Lagrange multiplier.

Name____ Mail Box ___________________

4) We have been assuming the following form for our discrete-time transfer functions with input $R(z)$ and output $Y(z)$

$$G_p(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Cross multiplying we get

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) = b_0 U(z) + b_1 z^{-1} U(z) + b_2 z^{-2} U(z)$$

In the time-domain this becomes

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_0 u(n) + b_1 u(n-1) + b_2 u(n-2)$$

Assume we have the following time-varying plant

$$G_p(z) = \frac{0.5z^{-1} + 1z^{-2}}{1 + 0.5z^{-1} + 0.1z^{-2}} \quad t < 2$$

$$G_p(z) = \frac{2z^{-1} + 1z^{-2}}{1 - 0.2z^{-1} + 0.3z^{-2}} \quad 2 < t < 4$$

$$G_p(z) = \frac{1z^{-1} - 1z^{-2}}{1 + 0z^{-1} - 0.3z^{-2}} \quad t > 4$$

We utilize a recursive least squares algorithm and get the results shown in the graph on the next page. Is the algorithm working or not? Explain you answer. If the algorithm is failing indicate all the ways you feel the algorithm is failing. If the algorithm is working, explain how the performance might be improved.

Name____ Mail Box ___________________