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# ECE-420
# Exam 2

# Fall 2012

**No Calculators allowed. You must show your work to receive credit.**

**Problem 1 __________/15**

**Problem 2 __________/30**

**Problem 3 __________/30**

**Problem 4 __________/25**

**Total ________________**

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1) Consider a 3 x 3 matrix A with eigenvalues 1,1 and 2. Assume we want to represent the function $e^{At}$ using the polynomial $\alpha_0 I + \alpha_1 A + \alpha_2 A^2$. Determine the three equations we need to solve in order to determine the $\alpha_i$. DO NOT SOLVE THE EQUATIONS.

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2) Consider the discrete-time state variable model

$$\underline{x}(k+1) = G\underline{x}(k) + Hu(k)$$

with the initial state $x(0) = 0$. Let

$$G = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0$$

a) Determin the characteristic equation for $G$

b) Show that we can write $x(4) = \begin{bmatrix} GH & H \end{bmatrix} \tilde{u}(3)$ and determine an expression for $\tilde{u}(3)$

c) Now assume we are using state variable feedback with $u(k) = G_{pf} r(k) - Kx(k)$. Assuming that $D = 0$ determine the transfer.

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3) For this problem, assume $\underline{p} = \begin{bmatrix} a \\ b \end{bmatrix}, \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, A = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ and we are **<u>given</u>** the following:

**i)** for $f(\underline{x}) = \underline{p}^T \underline{x}, \dfrac{df}{d\underline{x}} = \underline{p}^T$

**ii)** for $f(\underline{x}) = \underline{x}^T \underline{p}, \dfrac{df}{d\underline{x}} = \underline{p^T}$

**iii)** for $f(\underline{x}) = A\underline{x}, \dfrac{df}{d\underline{x}} = A$

**iv)** for $f(\underline{x}) = A^T \underline{x}, \dfrac{df}{d\underline{x}} = A^T$

**v)** for $f(\underline{x}) = \underline{x}^T A\underline{x}, \dfrac{df}{d\underline{x}} = x^T (A + A^T)$

**a)** The error vector $\underline{e}$ between observation vector $\underline{d}$ and the estimate of the input $\hat{\underline{x}}$ is $\underline{e} = \underline{d} - A\hat{\underline{x}}$. We want to weight the errors by a *symmetric* matrix $R$. Find $\hat{\underline{x}}$ to minimize $\underline{e}^T$ R $\underline{\mathrm{e}}$. (This is a weighted least squares.)

**b)** Assume we have the under determined system $\underline{y} = H\underline{u}$. Determine an expression for the value of $\underline{u}$ that solves this system of equations and has the minimum norm (minimizes $\underline{u}^T \underline{u}$). Hint: You will need to use a Lagrange multiplier.

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4) We have been assuming the following form for our discrete-time transfer functions with input $R(z)$ and output $Y(z)$

$$G_p(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

Cross multiplying we get

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) = b_0 U(z) + b_1 z^{-1} U(z) + b_2 z^{-2} U(z)$$

In the time-domain this becomes

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_0 u(n) + b_1 u(n-1) + b_2 u(n-2)$$

Assume we have the following time-varying plant

$$G_p(z) = \frac{0.5z^{-1} + 1z^{-2}}{1 + 0.5z^{-1} + 0.1z^{-2}} \quad t < 2$$

$$G_p(z) = \frac{2z^{-1} + 1z^{-2}}{1 - 0.2z^{-1} + 0.3z^{-2}} \quad 2 < t < 4$$

$$G_p(z) = \frac{1z^{-1} - 1z^{-2}}{1 + 0z^{-1} - 0.3z^{-2}} \quad t > 4$$

We utilize a recursive least squares algorithm and get the results shown in the graph on the next page. Is the algorithm working or not? Explain you answer. If the algorithm is failing indicate all the ways you feel the algorithm is failing. If the algorithm is working, explain how the performance might be improved.

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