

**ECE-420: Discrete-Time Control Systems**  
Homework 1

Due: Friday September 13 at the beginning of class

1) From class we have the useful sum  $S_N = \sum_{k=0}^{k=N} a^k = \frac{1-a^{N+1}}{1-a}$ . Using this sum, and possibly a change of variables, show that

$$\text{a) } \sum_{k=0}^{k=N} \left(\frac{a}{b}\right)^k = \frac{b^{N+1} - a^{N+1}}{b^N(b-a)}$$

$$\text{b) } \sum_{k=M}^{k=N} a^k = \frac{a^M - a^{N+1}}{1-a}$$

2) Starting from  $S_N = \sum_{k=0}^{k=N} a^k = \frac{1-a^{N+1}}{1-a}$ , take derivatives of both sides to show that

$$\sum_{k=0}^{k=N} ka^k = \frac{Na^{N+2} - (N+1)a^{N+1} + a}{(1-a)^2}$$

3) For impulse response  $h(n) = \left(\frac{1}{2}\right)^n u(n)$  and input  $x(n) = u(n)$ , show that the system output is

$$y(n) = 2 \left[ 1 - \left(\frac{1}{2}\right)^{n+1} \right] u(n) \text{ by evaluating the convolution sum } y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

*Note that this is the unit step response of the system.*

4) For impulse response  $h(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3)$  and input

$$x(n) = \left(\frac{1}{2}\right)^{n-1} u(n-2), \text{ determine the output } y(n) \text{ (this should be easy).}$$

5) Show that  $u(n) = \sum_{l=-\infty}^{l=n} \delta(l)$  and  $u(n-k) = \sum_{l=-\infty}^{l=n} \delta(l-k)$

6) For impulse response  $h(n) = \left(\frac{1}{3}\right)^{n-2} u(n-1)$  and input  $x(n) = \left(\frac{1}{2}\right)^n u(n-1)$ , show that the system

output is  $y(n) = 9 \left[ \left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1} \right] u(n-2)$  by evaluating the convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$$

7) For impulse response  $h(n) = \left(\frac{1}{2}\right)^{n-3} u(n-1)$  and input  $x(n) = \left(\frac{1}{4}\right)^{n+1} u(n-2)$ , show that the system

output is  $y(n) = \left[ \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^{n-1} \right] u(n-3)$  by evaluating the convolution sum  $y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$

8) For impulse response  $h(n) = \left(\frac{1}{5}\right)^n u(n)$  and input  $x(n) = u(-n)$ , show that the system output is

$$y(n) = \frac{5}{4} u(-n) + \frac{1}{4} \left(\frac{1}{5}\right)^{n-1} u(n-1)$$
 by evaluating the convolution sum  $y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$

9) For impulse response  $h(n) = \left(\frac{1}{3}\right)^{n-2} u(n-3)$  and input  $x(n) = \left(\frac{1}{2}\right)^{-n} u(2-n)$ , show that the system

output is  $y(n) = \frac{1}{5} 2^{n-2} u(5-n) + \frac{8}{5} \left(\frac{1}{3}\right)^{n-5} u(n-6)$  by evaluating the convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$$