**ECE-320**: Linear Control Systems
Homework 3

Due: Tuesday January 5 at the beginning of class.

**1)** (*sisotool problem*) For the plant modeled by the transfer function

$$G_1(s) = \frac{6000}{s^2 + 4s + 400}$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$PO \le 10\%$$
$$T_s \le 2.5 \text{ sec}$$
$$k_p \le 0.5$$
$$k_i \le 5$$
$$k_d \le 0.01$$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.

**2)** (*sisotool problem*) For the plant modeled by the transfer function

$$G_2(s) = \frac{6250}{s^2 + 0.5s + 625}$$

You are to design a PI controller, a PID controller with **complex conjugate zeros**, and a PID controller with **real zeros** that meet the following specifications

$$PO \le 10\%$$
$$PI\ T_s \le 15.0\ sec, PID\ T_s \le 0.5\ sec$$
$$k_p \le 0.5$$
$$k_i \le 5$$
$$k_d \le 0.01$$

In *sisotool*, in the LTI viewer, if you right click on the graph and select **Characteristics** you can let *sisotool* find the settling time. You should copy your step response and root locus plots to a word document, as well as including your values of the controller coefficients.

**3) (Easy)** Show that $\sum_{l=-\infty}^{l=n} \delta(l) = u(n)$ and $\sum_{l=-\infty}^{l=n} \delta(l-k) = u(n-k)$

**4) (Easy)** For impulse response $h(n) = \delta(n) + 2\delta(n-2) + 3\delta(n-3)$ and input

$x(n) = \left(\frac{1}{2}\right)^{n-1} u(n-2)$, determine the output $y(n)$ (this should be easy).

**5)** For impulse response $h(n) = \left(\frac{1}{2}\right)^{n} u(n)$ and input $x(n) = u(n)$, show that the system output is

$y(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$

a) by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$

b) by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{k=\infty} x(k)h(n-k)$

*Note that this is the unit step response of the system.*

**6)** For impulse response $h(n) = \left(\frac{1}{3}\right)^{n-2} u(n-1)$ and input $x(n) = \left(\frac{1}{2}\right)^{n} u(n-1)$, show that the system output is $y(n) = 9\left[\left(\frac{1}{2}\right)^{n-1} - \left(\frac{1}{3}\right)^{n-1}\right] u(n-2)$ by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$

**7)** For impulse response $h(n) = \left(\frac{1}{2}\right)^{n-3} u(n-1)$ and input $x(n) = \left(\frac{1}{4}\right)^{n+1} u(n-2)$, show that the system output is $y(n) = \left[\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{4}\right)^{n-1}\right] u(n-3)$ by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$

**8)** For impulse response $h(n) = \left(\frac{1}{3}\right)^{n+1} u(n-2)$ and input $x(n) = \left(\frac{1}{2}\right)^{n-2} u(n+1)$, show that the system output is $y(n) = \frac{16}{9}\left[\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{3}\right)^{n}\right] u(n-1)$ by evaluating the convolution sum $y(n) = \sum_{k=-\infty}^{\infty} h(n-k)x(k)$